Monetary Policy,Unanchored Inflation Expectation Risk and Government Bond Premia: An Indian Market Perspective

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Abstract

The [1] *paper examines the dynamics behind the persistent term premium (risk premium) of ten-year Indian Government Bonds, particularly under the current Inflation Targeting regime, with a particular emphasis on the drifts in the ten-year ahead inflation expectations. The Expectation Hypothesis channel of the ten-year bonds declined with the implementation of a lower inflation target of 4%, but the term premium component of these long-term bonds continues to fluctuate on average in the range of 110 basis points (bps) to 220 bps in both the pre and post-IT periods. This persistently large magnitude of term premium primarily arises from unanchored drifts in long-term inflation expectations (ten-year ahead) of the bond market participants, which occasionally get upward-revised during periods of counter-cyclical inflation. The underlying inflation in these periods exceeds substantially upper rational bounds of the participants with short-term inflation expectations. Such breaches recur frequently within a short span of five years, making participants hedge the risk a priori by charging a persistent term premium.*

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1 Introduction

The paper aims to explore the role of unanchored inflation expectations in explaining the observed persistent levels of nominal term premium in the long-term (ten-year) Indian Government Bonds, particularly under the current Inflation-Targeting (IT) regime, which commenced in 2015. The nominal term premium component consistently fluctuates between 110 and 220 basis points in both the pre and post-IT regimes (Figure 1).

The nominal term premium component represents the risk compensation that bond market participants require for holding long-term bonds until maturity. This compensation is generally linked to expectations around future probable deviations from the "Expectation Hypothesis Channel" during the bond's holding period, as discussed by Hördahl (2008) and Orphanides and Kim (2007).

Motivated by the unaltered persistent level of term premium observed in the Indian ten-year bond market, this paper aims to address four pivotal research questions :

(i) Have long-term (ten-year) inflation expectations in the Indian bond market remained consistently unanchored around the 4% target since the implementation of the IT regime?

(ii) To what extent do market participants display backward-looking behavior, indexing on historical inflation data for long-term expectations within the current IT framework?

(iii) What underlying periods primarily drive the unanchored inflation expectations of bond market participants?

(iv) Can unanchored inflation expectations sufficiently explain the premium levels observed in the long-term bond markets?

Figure 1: **The Ten-year Indian Government Bond Yield**

The Figure shows the Yields of the Ten-year Indian Government Bond for the period spanning from Q1:2010 to Q3:2021. The Risk-Free Yields and Term Premium components are extracted using the methodology of Adrian et al. (2015). The green and the orange lines depict the model prediction and the observed yields, respectively. The blue and purple lines depict the risk-free and the term premium components, respectively.

The paper begins with an analysis aimed at determining whether the long-term inflation expectations of participants in the bond market remain unanchored from the established 4% inflation target during the current inflation targeting period. Anchoring long-term inflation expectations is crucial for bondholders. Frequent unanticipated upward revisions in the long-term inflation expectations leads to significant risk by causing unexpected deviations in the expectation hypothesis channel. To mitigate risk with these unexpected deviations, bond market participants charge a term premium. Generally, the higher the stability, i.e., the more anchored the ten-year ahead (long-run) inflation expectations, the more muted will be the deviations in the expectation hypothesis channel leading to a low level of premium.

To assess the anchoring of long-term inflation expectations of the bond market participants, this paper begins by forecasting the ten-year-ahead inflation expectations of bond market participants using a novel Q-learning algorithm along the lines of Carvalho et al. (2023). This algorithm exclusively relies on the short-term inflation expectations of bond market participants and the observed inflation to generate the ten-year ahead forecast. The Q-learning algorithm is integrated with a New Keynesian DSGE model, where inadequately anchored long-term inflation expectations are the primary drivers of low-frequency fluctuations in agents' long-term beliefs about inflation. The extent to which these expectations are anchored is determined by the endogenous relationship between long-term expectations and short-term forecast errors. The strength of this relationship is influenced by the historical forecasting accuracy of the agents and the dynamics of Central Bank policy and structural shocks.

The ten-year-ahead inflation expectations generated using the algorithm up till March 2021-the end period of the study, are validated using ten-year forecasts solely available in the Survey of Professional Forecasters (SPFs) for a limited time period of Q3:2008 till Q1:2018. The forecasters in this survey comprise forty-two senior economists associated with the major bond market participants of India. The short-term inflation expectations used in the model are also sourced from SPFs.

The implemented methodology addresses the initial three critical research questions concerning the stability of long-term inflation expectations, as outlined in the introduction. It also helps identify periods that disrupt this stability, leading agents to become backward-looking and anchor to past inflation experiences when forming their long-run expectations

The algorithm operates within a framework where a Central Bank enforces a fixed inflation target and employs a discretionary monetary policy. This policy regime remains constant over time. Due to nominal rigidities, a subset of monopolistically competitive firms adjust their prices optimally each period based on their expectations regarding the future trends of marginal costs and inflation. These expectations depend on their beliefs about the long-term inflation target pursued by the Central Bank. The collective pricing behaviour of all the firms in the economy ultimately determines the actual inflation rate. This pricing mechanism influences the rational expectations of the representative agent in the model with inflation in both the short and long term. The SPFs, i.e., the bond market participants, in this case, are proxied by the representative agent in the algorithm. In the setup, all the sectors in the economy have symmetric expectation about the long-term inflation target assumptions.

In the framework, the agent's long-term expectations about inflation do not completely align with the target set by the Central Bank due to information asymmetry. The asymmetry mainly arises from the Central Bank's struggle to keep inflation close to its long-term target on a prolonged basis, or from inconsistent communication about the inflation target it aims for. Due to such asymmetry, the agents in the framework exhibit bounded rationality regarding short-term inflation expectations. This bounded rationality is based on their assumption of the long-term inflation target set by the Central Bank. Similar to Slobodyan and Wouters (2012), within this framework, the agent adjusts their beliefs about the long-term inflation target in response to short-term inflation surprises relative to their rational bounds. They continue to do this recursively until their beliefs about short-run inflation closely align with the underlying structural inflation process.

The recursive updating process utilizes a non-linear gain function that shifts between two forms: (a) **The Constant Gain Function**, in which the agent maintains a consistent weight on short-term surprises while recalibrating their expectations of long-term inflation targets, and (b) **The Decreasing Gain Function**, where the agents reduce the weight assigned to short-term surprises and increasingly relies on their prior expectations regarding the inflation target.

Recurrent inflation surprises exceeding an implicit rational bound based on the inflation target expectation make the agents predominantly adhere to the constant gain function, which reflects a backward-looking orientation. In this context, the agents internalize the aforementioned surprises via the constant-gain function as it updates its long-term inflation target expectation.

Conversely, during a protracted period of short-term inflation surprises that remain within rational bound, the agents exhibit heightened confidence and transition to a forward-looking stance regarding their long-term inflation target assumption. This transition catalyses a shift towards the decreasing gain function, progressively reducing the weight allocated to short-term inflation surprises. Ultimately, the shift towards a permanent adoption of the decreasing gain function indicative of the anchoring of long-term expectations depends critically on the Central Bank's ability to sustain inflation stability in proximity to its defined mean target over an extended time period.

The methodology employed in this paper is distinct from extant studies in India that have addressed the anchoring of inflation expectations using short-term inflation expectations. The studies have only focused on the behaviour of short-term inflation expectation of households or SPFs in contrast with the standard benchmark definition of the anchored inflation expectation involving long-run expectations as advocated by Bernanke (2007)[2].Before continuing

[2]The implemented framework utilizes a representative agent model, ensuring consistent results for all agents in the economy-both households and the Survey of Professional Forecasters (SPF). In India, the Central Bank does not conduct long-term inflation surveys (five years and beyond) for households. However, households and SPF's short-term inflation expectations here in India exhibit a parallel trend, with the former consistently surpassing the latter Das et al. (2019). This paper primarily emphasizes the SPF, given the specifics of the research question. Nevertheless, should the SPF expectations, which remain persistently at a lower bound, become unanchored–as illustrated in the subsequent sections of the paper–then the upper-bound expectations of households will also inevitably become unanchored.

further, this paper provides a general overview of existing research in India to highlight the shortcomings in these studies related to inflation anchoring. This overview underscores the significance of the framework introduced in this paper, which seeks to address crucial questions raised at the beginning of the introduction. Additionally, it emphasizes the importance of monitoring long-term expectations from an anchoring perspective, particularly for market participants such as bondholders, whose pricing of instruments depends on the stability of these long-term expectations.

For instance, Pattanaik et al. (2023) to demonstrate anchoring post-IT shows significant improvements in the properties like stability, consistency, and sensitivity of short-term expectations of representative agents (using household data) relative to realized inflation post-IT. These are quantified using partial correlation coefficients within an ordinary least squares (OLS) framework, wherein short-term expected inflation serves as the dependent variable and realized inflation surprises function as independent variables.

Analogously, using an OLS setup, Eichengreen et al. (2021) present evidence of decreasing realized inflation and short-term expected inflation among households and SPFs post-IT, accompanied by improvement in properties like reduction in the volatility of realized inflation, to support the notion of better inflation anchoring.

Similarly, using short-term household surveys, Asnani et al. (2019) provides evidence of improved anchoring of inflation expectations following the implementation of inflation targeting (IT). To highlight this improvement, the authors demonstrate limited spillover effects from non-core item inflation to both core and non-core short-term inflation expectations of households after adopting IT. They analyze qualitative responses from household surveys, which assess the consistency of responses in relation to realized inflation and the volatility of core and non-core items each quarter. The qualitative responses are categorized into five groups: (a) decline in prices, (b) no change in prices, (c) price increase less than the current rate, (d) price increase similar to the current rate, and (e) price increase greater than the current rate. The authors document in Table 2 of their paper, that approximately 31% of respondents shifted from category (e) to categories (a), (b), and (c), considering both non-core and core item inflation. They present this shift as evidence of improved anchoring, despite there being no changes in the volatility of non-core items post-IT.

In addition to the three aforementioned studies, research conducted by Goyal and Parab (2019) and Das et al. (2019) also provides evidence of a reduction in short-term inflation expectations in the post-IT period. Furthermore, Rangan and Das (2024) examined post-IT short-term inflation expectation data and concluded that expectations were effectively anchored until 2022, and these expectations subsequently became unanchored, which is implied by an increase in short-term expectations in their paper, in response to external shocks occurring in the same year.

Similarly, Goyal and Parab (2021) conducted a variance decomposition analysis of household short-term inflation expectations, covering both pre and post-IT periods. They found that shocks in the Central Bank's projections of short-term inflation expectations have the third-largest impact on households' short-term inflation expectations. This influence is surpassed only by shocks in households' own inflation expectations and shocks in food inflation.

The papers discussed above do not satisfactorily address the issue of anchoring and overlook the benchmark criteria of the long-term expectation stability established by Bernanke (2007). Firstly, none of the papers addressing the issue of inflation expectation anchoring in the Indian context features a structural setup. As explained in Jørgensen and Lansing (2025), testing for the anchoring through short-run expected inflation is feasible only when two conditions are strictly satisfied: a) long-run expectations are firmly anchored, and b) the slope of the New Keynesian Phillips Curve (NKPC) is relatively flat. In the Indian context, however, the NKPC has shown both upward and downward slopes in the periods before and after the implementation of the IT regime, respectively Patra et al. (2021). Only when conditions a) and b) are strictly upheld can anchoring through short-run inflation expectation be tested.

Secondly, the results discussed in the aforementioned Indian studies involving short-term expectations can also be characterized by a notable decline in headline inflation, often triggered by favourable exogenous shocks, similar to the sharp decrease in food and oil prices[3]observed in the post-IT period, as seen in the Figure A.4.1 and A.4.2. Such shocks typically lead to a decrease in both the volatility and persistence of short-term inflation expectations. Concurrently, agents may exhibit more optimistic qualitative responses in the short run, even when their inflation expectations lack strong anchoring in the long run.

Such evidence has been presented by Ascari et al. (2024) and the foundational work of Ball and Mankiw (1995). Large asymmetric price fluctuations within the headline inflation component–specifically, the first moment (mean) and the third moment (skewness)–play a significant role in influencing the endogenous short-run inflation expectations of agents and the associated uncertainty (volatility).

When the distribution of price changes shows skewness, an increase in variance amplifies the asymmetry present in the underlying inflation component. This phenomenon results in greater variability in the overall price level and short-run inflation expectations, suggesting that variance does not affect inflation and short-run inflation expectations in isolation. Rather, it interacts positively with skewness: higher variance is linked to high short-run inflationary expectations in the case of right-skewed shocks, while it can lead to deflationary short-run expectations when such shocks are left-skewed.

[3]Oil and Food Prices comprise around 60% of weight in the CPI index.

The pronounced decrease in headline inflation observed in the post-IT period, especially during the period of 2014-2016, is attributable to significant declines in oil, energy, and food prices, resulting in a marked negative skewness across the headline inflation components. This occurrence can elucidate various observed phenomena in the aforementioned studies, including a) downward revisions of short-term inflation expectations and subsequent reductions in their volatility; b) limited spillover effects from non-core to core inflation and expectations of non-core inflation subsequent to the implementation of inflation targeting; and c) the persistence of qualitative responses from economic agents, notwithstanding the absence of alterations in the second-order moment (variance) of the underlying inflation component.

Moreover, in instances where inflation expectations are effectively anchored, one should anticipate a more pronounced focus on the communications and projections of the Central Bank within the context of variance decomposition. This may be attributed to public trust in the communicated inflation target, which reflects the Central Bank's success in mitigating the aggregate effects stemming from inflationary shocks, as documented in the works of Grohé and Uribe (2024), Christelis et al. (2020), and Blinder et al. (2008).

The paper clearly distinguishes itself from the aforementioned studies here in India on the following lines:

(i) The implemented framework provides a coherent theory behind the formation and stability of long-term inflation expectations as per the benchmark definition of Bernanke (2007).

(ii) The stability of long-term inflation expectations relies primarily on short-term forecasts and realized inflation, as these are the only inputs to the framework. The rational expectational bounds with short-term inflation expectations and surprises with realized inflation, jointly influence the stability of long-term inflation expectations.

(iii) The framework addresses the issue of anchoring by implementing a model in which anchoring is tested through the stability of long-run inflation expectations conditional on short-term surprises.

The framework reveals that long-term inflation expectations have mainly declined due to the adoption of the 4% long-term inflation target by the Central Bank, leading to a significant downward revision in the inflation target during IT [4]. The long-term expectation has gradually reduced and converged within the Central Bank's official target band of 2%, and such reduction also corresponds with the decrease in the expectation hypothesis channel (Figure 1).

Nonetheless, concerns persist regarding the sustainability of long-term inflation at the established 4% target. The SPF consistently aligns with the continuous gain function, focusing on past short-term inflation surprises while adjusting their inflation target expectations. Such stickiness with constant gain is mainly driven by the significant inflation surprises faced by the agents in the immediate past.

In the established learning framework, the weight that the agent assigns to past short-term inflation surprises gradually decreases over time. Specifically, the weight assigned to surprises from two years ago is roughly 0.27, while the weight for surprises from five years ago is nearly negligible. It is only when these surprises remain within rational bounds for a duration significantly longer than the minimum five-year threshold that agents make a meaningful shift to decreasing gains.

This gradual decay helps to clarify why the SPFs here in India did not switch to a decreasing gain in the post-IT regime while updating their long-run expectations. Prior to the implementation of IT, the SPFs encountered significant inflationary shocks, which were positively skewed. As a result, in

[4] Anecdotal evidence suggests the Central Bank here in India was pursuing 8% target in the short-medium term and 6% target in the long-run before implementation of the IT regime.

the post-IT period, forecasters maintained a substantial weight on recent inflation surprises, even as they began to adjust their long-term expectations downward. These forecasters were still in the process of learning about the new regime shortly after IT was implemented while concurrently revising their long-term outlooks.

Similarly, within the current IT regime, in relation to the 4% mean target, the learning algorithm shows the rational bound of the agents within a range of 3.8% to 5.2%. The 5.2% to 3.8% range remains significantly below the upper limit of the official target set at 6% and above the lower limit of 2%. Over the six years following the implementation of IT, up to the end of the sample period in the paper, there are periods showing both significant upward and downward breaches of these short-term rational bounds, at times exceeding the limits set by the Central Bank. These fluctuations in the inflation process, which surpass the expected rational bounds, lead agents to adopt a constant gain approach in their updating rules even in the current IT regime.

On a larger time frame covering both pre and post-IT, most of the degradation in the gain function occurs during or around times of weak real consumption growth. The underlying breaches of inflation that lead to the deterioration of the gain function are historically counter-cyclical with respect to real consumption growth. In the current IT regime, these counter-cyclical exceedances have even surpassed the Central Bank's upper limit of 6%. The algorithmic assessment indicates that these counter-cyclical breaches recur within a span of five years, frequently exceeding the rational bounds the agents form with the short-term inflation conditional on their assumed long-run inflation target. These recurrent counter-cyclical breaches make the SPFs continuously backward-looking and stick to constant gain, making their long-term inflation expectations unanchored.

This situation clearly indicates why unanchored inflation expectations in India pose a significant long-term risk for bond market participants. As discussed in the works of Bansal and Shaliastovich (2013), Rudebusch and Swanson (2012), and Eraker (2008), bond market participants generally have an aversion to unexpected countercyclical inflation surprises. Such surprises lead to substantial upward revisions in the long-term inflation target assumptions of the participants, especially during periods of weak real consumption growth when the marginal utilities are high. Consequently, this results in unanticipated increases in yields during these vulnerable times. Simultaneously, the Central Bank's responses to these surprises have often been insufficiently aggressive, creating a challenging environment that participants did not anticipate when investing in long-term bonds.

Finally, the paper concludes with a short section to establish that unanchored expectations among participants in the Indian bond market lead to significant inflation risk premiums, which adequately account for the observed nominal premium levels in long-term bonds. When inflation expectations are unanchored, market participants tend to price premiums based on future expectations that reflect substantial short-term deviations in inflation from the target. These expectations primarily stem from the repeated large deviations in the realized inflation from the inflation target. The analysis shows that a considerable portion of the observed premiums can be attributed to these forward-looking expectations driven by unanchored beliefs, even in the current IT regime.

In the concluding section, to quantify nominal premiums and inflation risk premiums emerging from forward expectations that incorporate revisions in inflation target assumptions, this paper employs a Bayesian framework within the Long-Run Risk Consumption-based Asset Pricing models as proposed by Song (2017) and Bansal and Yaron (2004). The Bayesian approach introduces a backward-looking aspect to the framework. Within this consumption-based asset pricing framework, the representative agent incorporates a latent process with (i) a time-varying inflation target assumptions and (ii) a time-varying covariance structure between shocks in real consumption growth and the inflation target assumptions, determining the cyclicality of shocks-related to the inflation target.

The representative agent in this asset pricing framework also learns about the efforts made by the Central Bank to stabilize inflation around their target assumptions. Grounded in this learning from historical and contemporary economic data, the bond market participant, as represented by the agent, formulates forward expectations and subjective probabilities concerning the magnitude of anticipated shocks in inflation target assumptions over an extended horizon.

When these expectations are firmly anchored, the magnitude of anticipated shocks to the long-term inflation target remains low, resulting in subdued nominal premiums and inflation risk premiums. In the context of India, the agent primarily forms its forward expectations around a large magnitude of countercyclical adjustments to inflation target assumptions by other bond market participants. Such expectations are based on their learning about incoming inflation data, recurrent short-term inflation surprises, and the covariance with the state of real consumption growth. A significant probability associated with these expectations leads to persistently positive premium levels.

The paper contributes uniquely by establishing that unanchored inflation expectations, both before and after the introduction of IT in India, are the primary driver of persistent levels of term premiums. In the Indian context, key policy papers such as those by Patra et al. (2020) and Dilip (2019) highlight that international factors–such as global uncertainty, liquidity, and spillovers from the U.S.–are significant drivers of the term premium particularly in the post-IT period.

Additionally, the factors contributing to inflationary pressure may vary between international and domestic influences. For instance, an article by Sengupta and Vardhan (2021) illustrates how domestic fiscal channels have led to the steepening of yields following the implementation of IT. These policy papers are devoid of any structural setups and primarily engage in variance decompositions of channels behind short-term realized inflation and the resultant term premium.

Moreover, none of the mentioned papers engage in first-principle thinking regarding whether long-term inflation expectations are anchored and how much of the uncertainty around long-term expectations is factored into the premium levels through their variance decompositions. If inflation expectations remain anchored–an outcome that can only result from active inflation stabilization by the Central Bank–the levels of term premiums will likely be muted.

Relation to the Literature: The paper explains the factors contributing to the persistent level of the ten-year bond term premium. In addressing this question, the paper investigates how well-anchored long-term inflation expectations are among bond market participants.

The findings of this paper align with the existing literature, notably the work of Wright (2011) and Gürkaynak et al. (2005), which elucidates the transmission dynamics of short-term inflation surprises to long-term bond yields in the U.S. financial markets. Their studies demonstrate that a reduction in forward rates and nominal premiums is closely associated with the anchoring of inflation expectations.

Furthermore, this paper resonates with the analyses of Hördahl and Tristani (2014), Grishchenko and Huang (2012), and Ang et al. (2008), which reveal a consequential reduction in inflation risk premiums ranging from 14 to 19 basis points leading to a corresponding decrease in U.S. bond premiums during the pre-global financial crisis period.

In addition, the evidence presented herein complements the foundational research of Campbell (2009), Luis M. Viceira et al. (2009), Piazzesi and Schneider (2007), and Wachter (2006), suggesting that fluctuations in the correlation between inflation and macroeconomic indicators such as the output gap or consumption growth, coupled with monetary policy interventions, serve as a persistent source of nominal risk premiums in bond markets.

Lastly, this paper overlaps with the extensive literature addressing the role of non-rational or arbitrary beliefs in influencing inflation expectations, as articulated by Marcet and Nicolini (2003). Such arbitrary beliefs can culminate in misaligned short-term interest rate expectations, further contributing to the persistence of bond premiums, an issue highlighted in the research of Vàzquez (2024).

Structure of the paper: Section 2 of the paper focuses on addressing the questions around the unanchored inflation expectation in the post-IT period through the Q learning algorithm. Additionally, the section establishes the backward-looking behaviour and the high sensitivity to the short-term inflation surprises of the bond-market participants arising out of the unanchored long-term inflation expectations. Section 3 of the paper shows that the unanchored expectation of the bond market participants continues to drive their forward-looking expectations with large countercyclical revisions in the inflation target assumptions, leading to persistent premium levels priced in the long-term bond market.

2 Long-term inflation expectations

In this section, the Q-learning model implemented to test the stability of the long-term inflation expectations of the SPF is explained. The Q learning algorithm encapsulates a standard benchmark New-Keynesian model in which the Central Bank strictly implements the fixed inflation target, on which the necessary processes are augmented to elucidate the formation of long-term inflation expectations of the agents.

2.1 The Benchmark Model: Firm Price-Setting Problem

A continuum of monopolistically competitive firms have price-setting objective functions subject to Rotemberg 1982 type adjustment cost. Each i^{th} firm maximizes the discounted value of profits given by:

$$E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} M_{t,T}[Y_T(i)(\frac{P_t(i)}{P_T} - mc_t)] \tag{1}$$

Where $M_{t,T}$ is the stochastic discount factor, γ is the probability with which prices will be reset in each period, and β is the discounting factor. The demand curve that each i^{th} firm face is $Y_t(i) = (\frac{P_t(i)}{P_t})^{-\theta_{p,t}} Y_t$, wherein $\theta_{p,t}$ is the degree of the substitutions between the goods. $P_t(i)$ is the price set by the i^{th} firm given the overall price index P_t and mc_t is the real marginal cost. The optimal pricing decision of the firm, considering a fixed inflation target π^*, is expressed as:

$$P_t(i) = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t}[(1-\gamma\beta)(mc_t + u_T)] + \gamma\beta((\pi_{T+1} - \pi^*) - \gamma_p(\pi_T - \pi^*)) \tag{2}$$

γ_p is the degree of indexation with past inflation, and u_T is the cost-push shock. Log-linearizing the pricing strategy and assuming symmetric behaviour of all the firms, the aggregate supply curve is shown as:

$$\hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + \mu_t + E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t}[\tau_p \widehat{mc}_t + (1-\gamma)\beta(\hat{\pi}_{T+1} - \gamma_p \hat{\pi}_T)] \tag{3}$$

where, $\tau_p = (1-\gamma)(1-\gamma\beta)/\gamma$ and $\hat{\pi}_t$ is the log-linearized deviations from the steady state .

2.1.1 The Discretionary Monetary Policy

The Central Bank stabilizes the inflation around the steady inflation target, and the log-linearized inflation-targeting policy is of the discretionary form, as in Woodford (2001):

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} + \Gamma_x x_t = \epsilon_t \tag{4}$$

x_t is the output gap deviation, and the exogenous monetary policy shocks follow the AR(1) process: $\epsilon_t = \rho\epsilon_{t-1} + \xi_t, \text{with}, \xi_t \sim N(0,1)$

Marginal cost deviations are proportional to the output gap deviations $_t = \phi_t x_t$, and $\phi_t = 1$ is assumed for the closed-form solution of the Γ parameter and subsequent prior definition, which is defined in the subsequent section.

2.1.2 Rational Expectations of the Representative Agent

Given the price-setting behaviour of firms, under the benchmark model, the first-order log linearized rational expectation of a representative agent with a fixed inflation target is defined as:

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \tau_p \widehat{mc}_t + \beta E_t(\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t) + \mu_t \tag{5}$$

Combining (3) to (6), the log-linearized rational expectation with inflation can be further simplified by the stationary process defined as:

$$\hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + \rho\bar{\omega}\epsilon_{t-1} + \nu_t \tag{6}$$

where, $\bar{\omega} = \left[1 + (1-\beta\rho)\tau_p^{-1}\Gamma_x\right]^{-1}$ and $\nu_t = \bar{\omega}\xi_t + (1+\tau_p\Gamma_x^{-1})^{-1}\mu_t$

2.2 Arbitary Beliefs: The Perceived Law of Motion

The representative agent and the firms may doubt the resolve of the Central Bank in implementing a non-time-varying inflation target π^* and a time-varying inflation target deviation $\bar{\pi}_t$ (deviation from the non-varying steady state) is incorporated in their **Perceived Law of Motion (PLM)** while forecasting the inflation at time t-1. In the setup, both the firms and representative agents have symmetric expectations with deviations in the non-time varying inflation target $\bar{\pi}_t$. The rational expectation from equation (7) is modified to incorporate this time-varying inflation target $\bar{\pi}_t$ in the following manner:

$$\mathbf{PLM}: \hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + (1-\gamma_p)\bar{\pi}_t + \rho\bar{\omega}\epsilon_{t-1} + e_t \tag{7}$$

e_t is the forecast error.

2.2.1 The Actual Law of Motion

In the setup, firms derive their expectations about marginal cost and future inflation conditional on their assumptions about long-term inflation targets. The inflation target deviation assumptions are assumed symmetric. At any point in time, the belief structure of the agent regarding the long-run inflation target is shown in the log-deviation form:

$$\lim_{T\to\infty} \hat{E}_t \pi_T = \bar{\pi}_t \tag{8}$$

The anticipated belief about the long-run inflation at each time period is fixed to the inflation target, which agents think will prevail in the immediate future period. The expression in the log-deviation form is:

$$\hat{E}_{t-1}\hat{\pi}_T = \bar{\pi}_t \tag{9}$$

Such belief structure implies any revisions in inflation expectation solely arise from changes in expectations with the long-run inflation target. Given the expectation of the inflation target, the firms extrapolate the path of the future marginal cost. While making such extrapolation, the firms also take into account inertia in the monetary policy. The extrapolated marginal cost in the hat form can be defined as:

$$\hat{E}_t\widehat{mc}_{T+1} = \frac{1}{\Gamma_x}\hat{E}_t\left[\epsilon_{T+1} - (\hat{\pi}_{T+1} - \bar{\pi}_T) + \gamma_p(\hat{\pi}_T - \bar{\pi}_T)\right] \tag{10}$$

where the inertia in the monetary policy is expressed as $\hat{E}_t\epsilon_{T+1} = \rho^{T-t}\epsilon_t$

Given that each firm has arbitrary beliefs towards deviation from non-time varying inflation target, the true data generating process, i.e., the **Actual Law of Motion(ALM)** will have a fraction of firms incorporating such marginal-cost estimation while re-optimizing their price level following equation (1).

With a fraction of firms revising their price levels to maximize their profit with updated inflation target expectations, inflation is realized in the economy as per the ALM.

After accounting for the time-varying deviation in the inflation target, the rational expectation equation (7) of the representative agent is updated, resulting in the ALM of the form:

$$\mathbf{ALM}: \hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + (1-\gamma_p)\Gamma\bar{\pi}_t + \rho\bar{\omega}\epsilon_{t-1} + \nu_t \tag{11}$$

$$\text{where, } \Gamma = \frac{1}{1+\tau_p\Gamma_x^{-1}}\frac{(1-\gamma)\beta}{1-\gamma\beta}$$

In the framework, at an aggregate level, the divergence between the ALM and PLM mainly arises from the term Γ featuring in the ALM. The ALM and PLM differ due to the term $\Gamma\bar{\pi}_t$, where $\Gamma < 1$. The term Γ determines the degree of feedback from the arbitrary belief around inflation target deviations $\bar{\pi}_t$ to the realised inflation $\hat{\pi}_t$.

With strict inflation targeting, the weight on the output gap term Γ_x reduces consistently in the discretionary monetary policy of the Central Bank, further reducing the Γ term, i.e, feedback from the inflation target deviations to the realized inflation becomes muted. The difference between ALM and PLM reduces within a tight bandwidth. The inflation realized as per the ALM process is correctly perceived by the agents to a high degree of accuracy using their PLM process. At the limiting case $\Gamma \to 0$, both ALM and PLM converge to rational expectation with a steady-state inflation target as defined in equation (7), and the agents become forward-looking.

However, with more preference for output gap stabilization, the weight on Γ_x term increases. Consequently, the term Γ weight on the inflation target deviation $\bar{\pi}_t$ increases in the ALM. The difference between the ALM and PLM starts increasing and breaches a threshold, with the Central Bank taking more preference for output gap stabilization over strict inflation targeting.

At the limiting case as $\Gamma_x \to \infty$ or $\Gamma \to 1$, the PLM always diverges from ALM. As the difference between ALM and PLM increases beyond bound, the representative agent updates their time-varying inflation target $\bar{\pi}_t$ expectations. The update continues until both PLM and ALM are consistent with each other, and the agent again becomes structurally aware of the underlying inflation process. In such cases, the agent becomes backwards-looking and continuously updates their time-varying inflation target $\bar{\pi}_t$ deviation assumptions to remain structurally aware.

2.3 Inflation Targeting Update Rule

The inflation target deviation assumption is updated in each period using the switching gain function:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + g_{t-1}^{-1} e_{t-1} \tag{12}$$

where the term g_t is the gain function. As the realised inflation remains within a bound for a prolonged period, the agents become confident about the underlying structure of inflation and reduce their weight on the short-term forecast errors e_{t-1}. The weights are reduced continuously through decreasing gain function as outlined below in equation (14), i.e., the agent becomes forward-looking. $\hat{E}_{t-1}\hat{\pi}_t$ is the agent's one period ahead forecast using PLM and $E_{t-1}\hat{\pi}_t$ is the forecast from ALM. MSE $= E[\hat{\pi}_t - E_{t-1}\hat{\pi}_t]^2$, and Θ determines the bound on the rationality of the agents.

The right-hand side in equation (14) can also be interpreted as a signal-to-noise ratio. When the ratio remains within a bound, the agents are confident about their inflation target assumptions, and their weight on the short-term forecast errors is reduced following the decreasing gain function outlined in (14). With the ratio within the bound for a prolonged period, the agents become forward-looking and reduce their weight on the short-term forecast errors drastically; expectations anchor around the assumed inflation target assumptions, and pass-through of short-term inflation surprises towards any revisions in the inflation target and realized inflation through equation (12) and (13) is muted.

$$g_t = \begin{cases} g_{t-1} + 1, & \text{if } \mid \hat{E}_{t-1}\hat{\pi}_t - E_{t-1}\hat{\pi}_t \mid \leq \Theta * \sqrt{MSE} \rightarrow \textbf{Decreasing gain} \\ \bar{g}^{-1}, & \text{otherwise} \rightarrow \textbf{Constant Gain} \end{cases} \tag{13}$$

However, with frequent or a prolonged period of the signal-to-noise ratio exceeding the right-hand side bound in the equation (14), the agents are backward-looking and switch to constant gain. The agents have arbitrary beliefs about the inflation target, and to be structurally aware of the inflation process, the agents continue to put a constant weight on the recent forecast error while updating their inflation target deviation assumptions. The expectations are not fully anchored and are subject to frequent revisions following equation (13).

The difference between expected inflation from ALM and PLM in the right-hand side of the equation (14) can be expanded as:

$$\begin{aligned} \mid \hat{E}_{t-1}\hat{\pi}_t - E_{t-1}\hat{\pi}_t \mid &= \mid (1-\gamma_p)(\Gamma - 1)\bar{\pi}_t \mid \\ &= \mid (1-\gamma_p)(\Gamma - 1)(\bar{\pi}_0 + \sum_{T=0}^{t} g_T^{-1} e_T) \mid \end{aligned} \tag{14}$$

One can see that frequent large short-term forecast errors realized in the previous periods drive the signal-to-noise ratio above the rationality bound in the current period, leading to significant pass-through of the current short-term forecast errors in the updates of the inflation target. In the following section, it will be demonstrated how short-term inflation surprises that exceed an implicit threshold are affecting the signal-to-noise ratio, pushing it beyond rational limits. This is resulting in unanchored expectations within the current IT regime.

The derivations of the structural parameters and the detailed representation of the reduced set of equations are shown in the appendix A.1 and the appendix A.2. The reduced set of log-linearized equations wherein the state variables $\mathcal{E}_t$ represented by $\mathcal{E}_t = A_t\mathcal{E}_{t-1} + B_t\eta_t$ are linked to the observed variables in the level forms in the following manner:

$$Y_t = \begin{bmatrix} \pi_t \\ E_t^{SPF}\pi_{t+1} \\ E_t^{SPF}\pi_{t+2} \\ E_t^{SPF}\pi_{t+3} \end{bmatrix} = \pi^* + C_t^{'}\mathcal{E}_t + D_t o_t \tag{15}$$

The measurement errors for all the observed variables are the vector o_t. The matrices C_t and D_t are time-varying to account for missing observations. π^* represents the long-run inflation(inflation target) and is factored in the measurement equations to map with the observed variables, which are in the levels. $E_t^{SPF}\pi_{t+1}$, $E_t^{SPF}\pi_{t+2}$, & $E_t^{SPF}\pi_{t+3}$ are the SPF inflation forecasts for one, two, and three quarters ahead respectively.

2.4 Methodology used for estimation purpose

The Marginal Particle Filter, as proposed by Schön et al. (2005), is implemented for the Mixed Linear and Nonlinear State-Space model to effectively address non-linearity in the data. The system of reduced equations, detailed in Appendix A.2, is divided into nonlinear states $(\bar{\pi}_t, g_t)$ and linear states (π_t, e_t, s_t). The nonlinear states are addressed using the particle filter, while the linear states, conditional on the nonlinear states, are marginalized out and solved with the Kalman filter.

Bayesian updates are employed to estimate the model parameters, and the Metropolis-Hastings algorithm, which follows a coercive acceptance rate as outlined by Vihola (2012), is applied at the mode estimated using the BFGS algorithm described by Sims (1999). Additionally, importance sampling is carried out following the methodologies established by Andrieu et al. (2010)

and Doucet et al. (2001).

Due to the limited availability of survey data, the US posterior distribution is adopted as the prior for all parameters to establish appropriate bounds for the rationality parameter Θ, except for the average inflation, exogenous shocks, and observation errors. The likelihood is adjusted using a scaling parameter λ to ensure that the model parameters remain close to the US posteriors while also maintaining rationality bounds and capturing sample-specific information. The scaling parameter λ is set to 0.5.

$$P^{In}(\bar{\Delta}^{In}|Y_t^{In}, Y^{US}, \bar{\Delta}^{US}) = L(Y_t^{In}|\bar{\Delta}^{US}, \bar{\Delta}^{In})^{\lambda} L(Y_t^{US}|\bar{\Delta}^{US}) p(\bar{\Delta}^{US}) p(\bar{\Delta}^{In})$$

The source of inflation and short-term inflation expectation data are defined in the appendix A.3.

2.5 Discussion: Parameter estimations and Unanchored Inflation Expectation

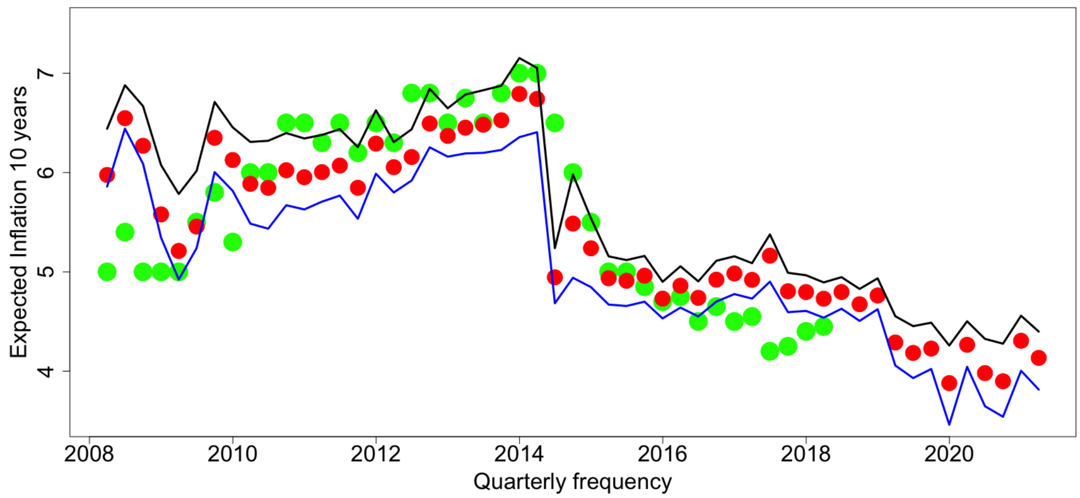
The priors and posteriors of the learning model are presented in Table 1. The posteriors are estimated at the mode, and the overall mean acceptance rate is 21.4% after running for 100,000 iterations, with a burn-in period of the first 50000 iterations. The steady-state inflation parameter π^* is set with a prior mean of 6.35% and a standard deviation of 0.35% for the pre-IT regime and a prior mean of 4.35% with the same standard deviation for the post-IT regime. These priors encompass the trend estimates obtained using the Unobserved Components Stochastic Volatility (UCSV) model developed by Stock and Watson (2007). During the sample period analyzed, the average inflation rate in the pre-IT regime was nearly 7%, while in the post-IT regime, it was approximately 5%. The algorithm effectively reproduces the ten-year-ahead inflation expectations observed in surveys (Figure 2). The tight bounds associated with the rationality parameter θ allow us to capture the evolution of long-run inflation expectations, conditional on short-term forecast errors.

Table 1: **The Priors and the Posterior distribution of the Q-learning algorithm**

	Prior			Posterior		
Para	Dist	Mean	Std	Mode	Mean	Std
$\pi^*_{post_{IT}}$	Normal	4.35	0.10	4.44	4.49	0.018
$\pi^*_{pre_{IT}}$	Normal	6.35	0.10	6.19	6.20	0.012
θ	Gamma	0.022	0.006	0.029	0.0218	0.005
$\bar{c}$	Gamma	0.126	0.028	0.127	0.147	0.007
Γ	Beta	0.906	0.041	0.872	0.833	0.02
ρ_s	Beta	0.879	0.028	0.861	0.843	0.011
γ_p	Beta	0.140	0.029	0.405	0.409	0.009
σ_s	IGamma	0.5	10	1.268	1.267	0.0126
σ_μ	IGamma	0.5	10	1.032	1.034	0.007

At this stage of the discussion, the paper revisits the pivotal question raised in the introduction: whether long-term expectations of agents have remained unanchored since the implementation of the IT framework. What specific periods significantly contribute to this unanchoring of long-term expectations? The analysis indicates that a substantial portion of the decline in long-run inflation expectations following the adoption of IT can be attributed to the lower inflation target established by the Central Bank. While long-term expectations derived from SPFs have increasingly aligned within the official target range, they have yet to stabilize consistently within a narrow bandwidth

Figure 2: **The Ten Year Ahead Expected Inflation**

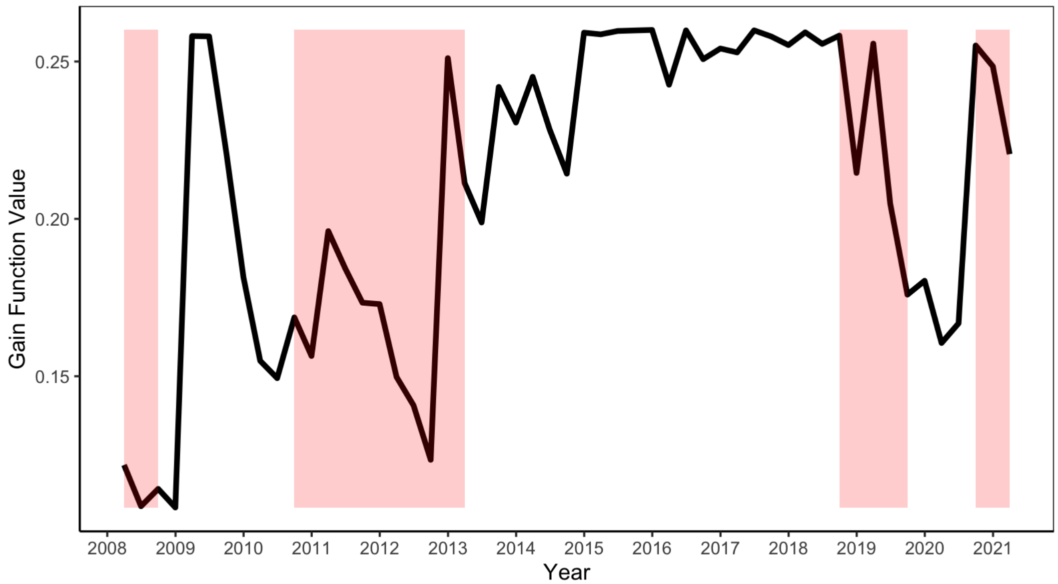


The Ten-Year-Ahead Expected Inflation is shown for the period spanning from March 2008 to March 2021. The green dots denote the expected inflation rate ten years into the future, and are sourced from the Survey of Professional Forecasters, which is available for the duration of March 2008 to March 2018. The red dots represent the model median estimates. Additionally, the black and blue lines illustrate the 97.5% confidence interval.

around the mean target of 4% (see Figure 2). Participants continue to adopt a backward-looking perspective, placing considerable emphasis on short-term inflation surprises, as demonstrated by the gain function (refer to Figure 3).

Given the posterior values of the constant gain weight $\bar{g}_t$ within the framework, the weight assigned to two-year-old short-term surprises is nearly 0.27, while the weight on five-year-old and above short-term surprises is near zero. Such a weighting scheme helps explain why, in the immediate years following the implementation of IT with a lower inflation target, long-run expectations decreased promptly yet remained unanchored within a narrow bandwidth around the mean target of 4%. The gradual decline in long-term inflation expectations is primarily attributed to the immediate downward revisions of the inflation target to 4%. However, given the significant breaches that occurred in the immediate past prior to the implementation of IT (Figure 4),

Figure 3: **The Gain Function**

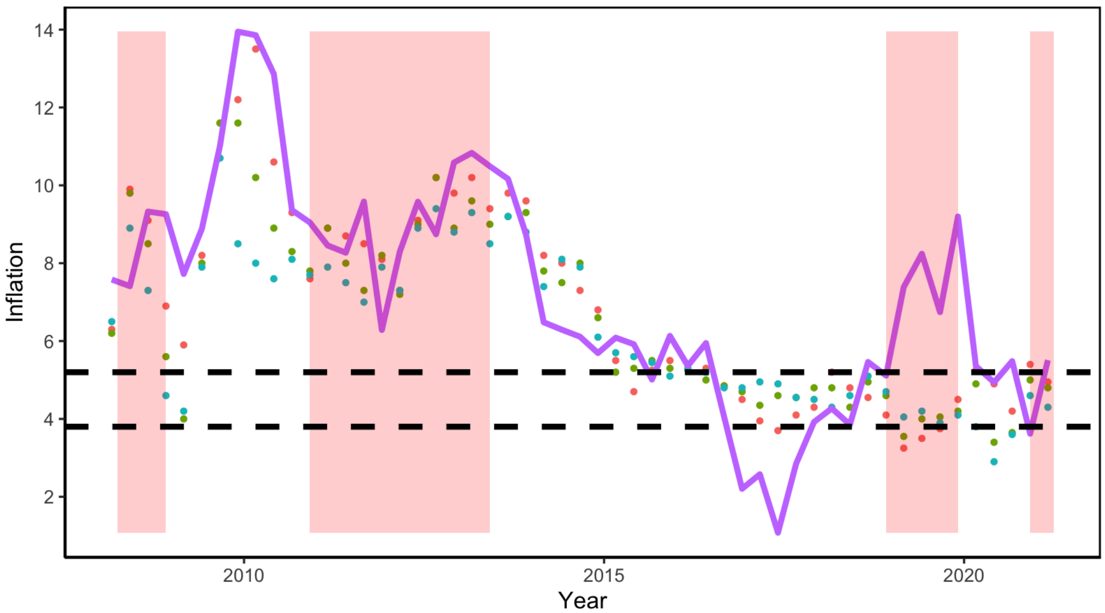


The black line illustrates the gain function generated for the SPF during the period from March 2008 to March 2021. The shaded area represents the OECD recession indicators for India.

agents continued to assign considerable weight to short-term inflation surprises while downward revising their long-term expectations in the immediate post-IT periods.

Now, in the current IT regime, significant breaches beyond the rational bounds continue to occur, making agents consistently stick to constant gain function, i.e., making them backwards-looking. Under the current regime with a target of 4%, the implicit rational bounds range indicates active inflation stabilization within 3.8% to 5.2%, which is notably tighter than the 2% confidence interval followed by the Central Bank. Within the limited sample period considered in the post-IT, two consecutive breaches of these rational bounds have already taken place (Figure 4), and similar breaches, mostly in the upside, have repeatedly occurred in past regimes.

Figure 4: **The Realized Inflation Vs the Short-Term Inflation Expectations**



The realized inflation is depicted by the bold line. Inflation expectations for one, two, and three quarters ahead, sourced from the Survey of Professional Forecasters, are represented by orange, green, and blue dots, respectively. The plot covers the period from March 2008 to March 2021. The dotted line indicates the rational bounds of 3.8% to 5.2%, corresponding to the 4% ± 2% inflation target framework implemented in 2015. The shaded area shows the OECD recession indicators for India.

Furthermore, as illustrated in Figure 4 and the gain function depicted in Figure 3, it is evident that the majority of the historical breaches in India, both in pre and post-IT regime, particularly those exceeding rational bounds, are predominantly skewed towards the upside. These breaches tend to manifest during periods that coincide with or follow phases of moderating real consumption growth. Such significant deviations indicate the Central Bank's heightened focus on stabilizing the output gap within its discretionary monetary policy framework, resulting in notable inflationary surprises during times of declining real consumption growth. Additionally, these countercyclical breaches appear to recur approximately every five years, playing a crucial role in driving unanchored long-term expectations.

From the perspective of long-term bondholders, these repeated countercyclical breaches create a persistent source of risk in the long-term bond market due to unanchored expectations among the bond market participants. With unanchored expectations, individual bond market participants assign a substantial probability to the possibility of significant future upward revisions of inflation target assumptions by other bond market participants. As the rational bounds are breached, these market participants upward revise their inflation target assumptions via the mechanism indicated in equations (13) and (14). This upward adjustment in the inflation target leads to unexpected increases in yields through an increase in the expectation hypothesis channel. Consequently, long-term bonds become loss-inducing instruments during periods of moderating real consumption growth.

Given the repetition of these breaches, particularly within a five-year timeframe, ten-year ahead expectations remain unanchored for bond market participants. This uncertainty leads agents to question the consistent maintenance of inflation within the rational bounds conditional on a specific long-term inflation target assumption, particularly during times of high marginal utility. As a result, they begin to hedge preemptively against these unforeseen upward revisions in inflation targets by incorporating a risk premium into long-term bond pricing.

This pattern of recurring events aligns with the points discussed at the outset of the introduction, underscoring unanchored expectations as the primary driver of persistent risk premiums in the long-term Indian bond market.

In conclusion, this paper establishes that unanchored forward-looking expectations are driving significant inflation risk premiums, which contribute to the overall nominal premiums. The nominal premium comprises both the inflation risk premium and real premiums. To analyze this, an asset pricing model is implemented that jointly accounts for changes in forward expectations with the inflation target assumptions and real consumption growth. This framework helps explain how the previously discussed recurrent changes in the assumptions about the inflation target, which are correlated with real consumption growth, are reflected in bond pricing. Specifically, forward-looking expectations tied to inflation target revisions are the main drivers of the inflation risk premiums. The analysis reveals that the average inflation risk premium is approximately 160 basis points, accounting for the majority of the average observed term premium of 198 basis points in the ten-year Indian bond market.

To illustrate this forward-looking mechanism for the term premium estimation, the Bayesian update within the Long-Run Risk Consumption-based Asset Pricing frameworks is implemented in the lines of Song (2017) and Bansal and Yaron (2004). The Bayesian approach introduces a backward-looking aspect to the framework. Conditional on incoming economic data, the bond market participant, represented by a representative agent, forms forward expectations and subjective probabilities with future revisions in inflation target assumptions, along with real consumption growth. In alignment with prior discussions, representative agents in the framework have a latent process with respect to a) inflation target assumption and additionally, b) a time-varying covariance structure between shocks in real consumption growth and the inflation target assumptions, determining the cyclicality of shocks to the inflation target assumptions.

Utilizing incoming data on real consumption growth and inflation, agents figure out shocks within these latent processes. Similar to the previous discussion in the paper, the shocks in the inflation target can exhibit either procyclical or countercyclical covariance relative to shocks in real consumption growth. Throughout this process, the representative agent continuously evaluates the inflation stabilization policies implemented by the Central Bank based on their revised assumptions regarding inflation targets.

These underlying interactions lead agents to derive transition probabilities with future adjustments to inflation target assumptions in conjunction with real consumption growth. In the Indian context, it is observed that bond-market participants perceive the countercyclical shifts in the inflation target assumptions as an absorbing state, both before and after the adoption of the IT framework. Furthermore, the bond market participants continue to expect a large magnitude of countercyclical revision even under the current IT regime.

The magnitude and persistence of the premium, which is the function of forward expectations in this asset pricing framework, depend on the frequency and intensity of countercyclical inflation deviations from the inflation targets experienced by the bond-market participants; the larger the inflation deviations, the more enduring the high levels of premiums will be, especially in an environment of unanchored expectations.

In the next section, the setup of the Bayesian consumption-based long-run risk asset pricing model is briefly explained. Readers at this stage can proceed to Section 3.4 for the results on the Bond pricing moments, transition probabilities and inflation-risk premium estimates.

3 How much risk-premium Ten-year bonds command given unanchored inflation expectations?

The representative agent has an Epstein-Zin type recursive preference like in Epstein et al. (2013) and Bansal and Yaron (2004) and maximizes its lifetime utility from the consumption stream:

$$U_t = max_{C_t}((1-\delta)C_t^{\frac{1-\gamma}{\kappa}} + \delta(E_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\kappa}})^{\frac{\kappa}{1-\gamma}} \tag{16}$$

The agent in the setup is endowed with an efficient market portfolio in which its entire wealth is invested. In each period, the agent consumes a fraction of the evolved wealth as C_t while the remaining wealth remains invested at an expected return.

In this recursive process of maximizing its utility, at each point in time, the agent forms forward expectations with the shocks in the key latent processes driving its pricing kernel. The expectations with the shocks are conditional on the incoming economic data related to the key latent processes.

The agent resolves these expected shocks early by pricing premiums in the assets, which are part of its efficient portfolio, at a discount. γ is the relative risk aversion and $\kappa = \frac{1-\gamma}{1-1/\psi}$ wherein ψ is the intertemporal elasticity of substitution(IES), with $\gamma > 1/\psi$ due to early resolution of the risk by the agents.

3.1 The key latent processes

The long-run risk in the framework resulting in the risk premium levels in the assets arises from the forward-looking expectation the agent forms around a) the inflation target assumption and, additionally, b) a time-varying covariance structure between shocks in real consumption growth and the inflation target assumptions. The agent estimates the expected magnitude and direction of shocks in these latent components based on its learning of incoming and past economic data

3.1.1 The real consumption latent process

The agent estimates the persistent trend in real consumption by observing the real consumption growth data. The real consumption growth, expressed as $\log \frac{c_{t+1}}{c_t}$, is divided into persistent latent processes $y_{c,t}$ and idiosyncratic components.

$$
\begin{aligned}
log\frac{c_{t+1}}{c_t} &= \mu_c + y_{c,t} + \sigma_c \eta_{c,t+1}, where \;\; \eta_{c,t+1} \sim N(0,1) \\
y_{c,t} &= \varphi_c y_{c,t-1} + \sigma_{y,c,t} \xi_{y,c,t}, where \;\; \xi_{y,c,t} \sim N(0,1)
\end{aligned} \tag{17}
$$

Shocks in the persistent latent process $\sigma_{y,c,t}\xi_{y,c,t}$ take time to dissipate and influence the trajectory of future real consumption growth before the persistent process mean reverts to its original trend. These shocks serve as the source of the real risk premium in assets due to their persistent nature. To enhance the model training for better identification of the shocks and trends in the real consumption growth, we also include the aggregate real dividend growth as a proxy for the equity market. The real dividend growth rate has a leveraged exposure to $y_{c,t}$, with the leverage controlled by the parameter ϕ[5].

$$
log\frac{d_{t+1}}{d_t} = \mu_d + \phi y_{c,t} + \sigma_d \eta_{d,t+1}, \;\; where \;\; \eta_{d,t+1} \sim N(0,1) \tag{18}
$$

μ_c and μ_d are the mean growth rate in the real consumption and real dividend, respectively.

3.1.2 The nominal component of long-run risk framework

By analyzing inflation data alongside real consumption growth data, agents seek to estimate shocks and revisions in the inflation target assumptions. As noted in the previous section, breaches of the rational bound in the learning model primarily occur during periods of weak real consumption growth. This observation motivates the incorporation of the covariance term in the latent inflation target path with respect to real consumption growth.

$$
y_{\pi,t} = \rho_\pi y_{\pi,t-1} + \sigma_{y,\pi,t}\xi_{y,\pi,t} + \alpha(S_t)\sigma_{y,c,t}, where \;\; \xi_{y,\pi,t} \sim N(0,1) \tag{19}
$$

[5]The rationale behind this leverage is that if overall real consumption growth follows a persistent trajectory, then the aggregate earnings and net worth of firms will evolve in multiple with this consistent real consumption growth rate. Consequently, the growth of aggregate dividends, which represent profits distributed by firms, will be proportional to this persistent growth rate.

The covariance term is denoted by $\alpha(S_t)$ and exhibits regime switching. This covariance can either be procyclical or countercyclical in relation to real consumption growth and cannot be zero. The agent seeks to reduce uncertainty and form forward expectations around future shocks to the inflation target assumptions and the covariance term, which is conditional on the incoming data. Similar to the real shocks, these shocks to inflation target assumptions also take time to dissipate and influence the trajectory of future inflation. This dynamic generates an inflation risk premium in assets.

Additionally, these estimated shocks to the inflation target assumptions contribute to the agents' endogenous inflation expectations. When forming their inflation expectations, the agent also considers the Central Bank's policy responses towards inflation stabilization around their inflation target assumptions. Consequently, the inflation expectations reflected in inflation risk premiums depend on the forward expectation with the cyclicality of shocks to the inflation target assumptions and the Central Bank's inflation stabilization policies around such assumptions.

The monetary policy is of the form:

$$\underbrace{i_t}_{Nominal\ rates} = \tau_0 + \underbrace{\tau_c y_{c,t}}_{real\ growth} + \underbrace{\tau_\pi(\pi_t - y_{\pi,t})}_{inflation\ around\ target} + \underbrace{y_{\pi,t}}_{Target} + \underbrace{y_{i,t}}_{Policy\ shock} \tag{20}$$

τ_0, τ_c, τ_π are estimated across all the regimes, and we let data speak about the nature of inflation stabilization seen by the agent. The policy shock, which is a latent state in the setup, follows an AR(1) process. The agent also makes prior resolutions with such shocks, which basically means to what degree the Central Bank can deviate from its traditional policy style to curb inflationary pressure or restore consumption growth.

The SDF, asset prices and returns, and inflation expectations are affine functions of these three latent state variables and shocks in these latent processes. The above latent states are represented in the reduced form:

$$Y_{t+1} = \Phi_1 Y_t + \Phi_2(S_{t+1})\Sigma_y(S_{t+1})\eta_{y,t+1}, \quad \eta_{y,t+1} \sim \mathcal{N}(0,I) \tag{21}$$

where, $Y_t = [y_{c,t}, \quad y_{\pi,t}, \quad y_{i,t}]^T$ and $\eta_{y,t} = [\eta_{y,c,t}, \quad \eta_{y,\pi,t}, \quad \eta_{y,i,t}]^T$

$$\Psi_1 = \begin{bmatrix} \varphi_c & 0 & 0 \\ 0 & \varphi_\pi & 0 \\ 0 & 0 & \varphi_i \end{bmatrix}, \Psi_2 = \begin{bmatrix} 1 & 0 & 0 \\ \alpha(St_t) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Sigma y = \begin{bmatrix} \sigma_{yc} & 0 & 0 \\ 0 & \sigma_{y\pi}(St_t) & 0 \\ 0 & 0 & \sigma_{yi} \end{bmatrix}$$

3.2 The Bond prices and Term Premium

The bond prices and yields are expressed in terms of the latent states and exhibit an affine relationship as illustrated below. Detailed derivations can be found in the appendix A.12. The nominal premium of the bonds is the function of the expected shocks in the latent states and the resultant expected bond yield loadings. The shocks in the latent states, which gradually decay over time, drive the bond yields(inverse of the price) and the asset premiums.

$$p_{n,t}(S_t) = C_{n,0}(S_t) + C_{n,1}(S_t)Y_t \tag{22}$$

As the shocks to the latent processes exhibit regime switching, for clarity, the relationship of bond yield loadings with the latent process regarding real consumption growth and the assumptions for inflation targets are presented within a particular regime.

$$\textbf{Real consumption growth loading:} -\frac{1}{n}C_{n,1,c} = \frac{1}{n}\frac{(1/\Psi\tau_\pi - \varphi_c\tau_c)}{(\tau_\pi - \varphi_c)}\frac{(1-\varphi_c^n)}{(1-\varphi_c)}$$

$$\textbf{Inflation target loading:} -\frac{1}{n}C_{n,1,\pi} = \frac{1}{n}\frac{(1-\varphi_\pi^n)}{(1-\varphi_\pi)}$$

The bond yield associated with inflation target assumptions is consistently positively correlated, regardless of the regime. When there are positive shocks–meaning upward adjustments–in inflation target assumptions, bond yields tend to increase. Conversely, yields decrease when there are negative shocks to these assumptions. This relationship explains the decline in expected yields of long-term Indian bonds following the downward revision of the inflation target to 4%(Figure 1).

The bond yield loadings on the latent path of real consumption growth within a particular regime are conditional on the Central Bank's inflation stabilization policy. In a regime where the Central Bank actively works to stabilize inflation, i.e., $\varphi_c < \tau_\pi$, loadings on real consumption growth latent states are positive, i.e., the yields are procyclical with respect to the real consumption growth.

However, in a regime where the Central Bank does not actively stabilize inflation, i.e.,$\varphi_c > \tau_\pi$, loadings on real consumption growth latent states become negative, i.e, the yields are countercyclical with respect to the real consumption growth.[6]

The Term Premium, as shown in the appendix A.14, is the average of the one-period expected excess return $rx_{n,t}$ that agents command for holding a bond of a particular maturity. This one-period excess return forecasted at l^{th} period ahead conditional on the prevailing regime $S_t = k \in (procyclical, countercyclical)$ is expressed as:

[6]Such movement of bond yields can also be better understood from the equities-bond correlation. Under active stabilization of inflation, with strong real consumption growth, equities become attractive over bonds in terms of high realized return. Market participants shift to equities from the bonds and the bond yield increases. The increased yield allows agents to use the bonds as a hedging instrument. Hedging is primarily done considering probable future periods of low real consumption growth, mostly arising from active inflation stabilization done by the Central Bank. However, in a high inflation regime arising out of the Central Bank's less preference towards inflation stabilization, both bonds and equities are risky due to inflationary risk and tend to move in the same direction. As strong real consumption growth is realized, the overall risk reduces, making both bonds and equities attractive for investment.

$$
\left(E_t(rx_{n,t+l}) + \underbrace{\frac{1}{2}Var(rx_{n,t+l})}_{Jensen\ \ Inequality\ \ Term} \quad |S_t = k\right)
$$

$$
\approx - \begin{bmatrix} p_{k,pro} & p_{k,coun} \end{bmatrix} \begin{bmatrix} p_{pro,pro} & p_{coun,pro} \\ p_{pro,coun} & p_{coun,coun} \end{bmatrix}^{l-1} \begin{bmatrix} \zeta(k,pro) \\ \zeta(k,coun) \end{bmatrix}
$$

where, $\zeta(k,j) \propto \Phi_2(j)\Sigma_y(j)\Sigma_y\Phi_2(j) \times \left(C_{n-1,1}(j)\right) \;\; with j \in (pro, coun)$[7]

where,

$$
\Phi_2 = \begin{bmatrix} 1 & 0 \\ \alpha(S_t) & 1 \end{bmatrix}, \Sigma_y = \begin{bmatrix} \sigma_{y,c} & 0 \\ 0 & \sigma_{y,\pi} \end{bmatrix}
$$

Agents form subjective probabilities and forward expectations regarding the potential regimes that may dominate in future periods based on their evaluations of incoming and historical economic data.

When agents are recurrently exposed to countercyclical regimes, they tend to place considerable weight on the transition probability $p_{coun,coun}$ in their forward expectations. Additionally, based on the Central Bank's monetary policy responses, the agent makes an estimate for the coefficients in the monetary policy response function, which may prevail in the future period.

The representative agent commands a positive excess return as they form a significant likelihood of a countercyclical regime, indicated by a high $p_{coun,coun}$ in their forward expectation. Further, agents form expectations with bond yield loadings $C_{n-1,1} < 0$, with a monetary policy that is less focused on active inflation stabilization.

The level of this excess return also depends on the magnitude of the shocks $\Phi_2(j)\Sigma_y(j)\Sigma_y\Phi_2(j)$ which agents incorporate in their forward expectation with the latent process. This magnitude of the innovation is conditional on the short-term inflation surprises to which the agents are recurrently exposed. If

[7] pro implies procyclical, con implies countercyclical

Table 2: **The Priors and the Posterior distribution of the Asset Pricing Framework**

Para	Prior			Posterior		
	Dist	Mean	Std	Mode	Mean	Std
$\alpha(+)$	U	3.27	1.6	1.15	1.15	2e-6
$\alpha(-)$	U	-3.27	1.6	-2.4	-2.4	1e-6
τ_0	G	0.0066	0.0054	0.0094	0.0094	3e-6
τ_π	G	1.14	0.50	0.86	0.86	2e-6
τ_c	G	0.21	0.097	0.32	0.32	3e-6
$\sigma_{y,c}$	IG	0.0005	0.0002	0.00119	0.00119	2.2e-9
$\sigma_{y,\pi,PP}$	IG	0.0009	0.0002	0.0003	0.0003	7.8e-11
$\sigma_{y\pi,CP}$	IG	0.0009	0.0008	0.00413	0.00413	3.5e-11
$\sigma_{y,i}$	IG	0.0009	0.0001	0.00143	0.00143	1.3e-9
$p_{pro,pro}$	Beta	0.46	0.20	-	0.38	0.134
$p_{coun,coun}$	Beta	0.55	0.23	-	0.97	0.011

the long-term inflation expectations are firmly anchored, which solely depends on the inflation stabilization policies of the Central Bank, the magnitude of innovations in their forward expectations will be low, and the levels of the nominal premium will be muted.

3.3 Estimation Methodology used for the asset pricing model

The sequence of observations used for the measurement equations is as follows:

$$Y_t = (log(\frac{c_t}{c_{t-1}}), \pi_t, pd_t, y_{1,t}, y_{2,t}, y_{5,t}, y_{10,t})$$

The variable pd_t implies log price to dividend and is connected to dividend growth following the methodology outlined in Campbell and Shiller (1988). Both the price and dividend are estimated per share basis. $y_{1,t}, y_{2,t}, y_{5,t}, y_{10,t}$ represents the ZCYC yields of one, two, five and ten-years respectively. Details of data sources are shown in the appendix A.5.

The posterior update is implemented at the estimated mode with a Metropolis-within-Gibbs sampler following Carter and Kohn (1994) and Kim and Halbert (1999), with a coerced acceptance rate of Vihola (2012). Simulations are done 2.5 million times, achieving a mean acceptance rate of 22.04%. The source of the data and priors for the fixed parameters are defined in the appendix A.15. The priors and posteriors for the remaining parameters are defined in the Table 2

3.4 Discussion: Parameter, Term Premium Estimation and Counterfactual

The model closely reproduces the moments observed in bond yields and term premiums in both the pre and post-IT regimes, shown in Table 3 and 4, respectively. The posteriors in Table 2 indicate that agents, while forming their forward expectations, assign a significant transition probability (p22)towards countercyclical revisions and a high magnitude of covariance term($\alpha(-)$) is embedded in their future inflation target assumptions, triangulating the findings of the unanchored expectations. While forming their forward expectations, the agents also internalize the Central Bank's less preference towards

Table 3: **The Model vs Data asset pricing moments**

	Data		**Model**	
Variable	Mean	Std	Mean	Std
$y_{1,t}$	6.56	1.44	6.69	1.36
$y_{2,t}$	6.8	1.21	6.79	1.36
$y_{5,t}$	7.26	0.88	7.08	1.37
$y_{10,t}$	7.63	0.78	7.40	1.37
r_m	13.6		13.85	
$\sigma(rm)$	0.26		0.24	
$acf(y_{10,t})$	0.67		0.65	
$Termpremium$	198bps		194bps	

The table presents a comparative analysis of the model and data asset pricing moments for one-year ($y_{1,t}$), two-year ($y_{2,t}$), five-year ($y_{5,t}$), and ten-year ($y_{10,t}$) zero-coupon yield curves (ZCYC) and the aggregate equity market returns. This analysis is conducted over the sample period spanning from July 2005 to December 2021.

active inflation stabilization, as evident from the posterior estimate of τ_π in the monetary policy response function. The estimate of the monetary policy coefficient is also in line with the documented passive policy response of the Indian Central Bank observed in Eichengreen et al. (2021) and Goyal and Tripathi (2014).

Given such forward expectations, agents consistently price in a persistent magnitude of premium both in pre and post-IT periods. The autocorrelation

Table 4: **The Model vs Data asset pricing moments post-IT period**

	Data		**Model**	
Variable	Mean	Std	Mean	Std
$y_{1,t}$	5.66	1.3	5.85	1.002
$y_{2,t}$	5.99	1.15	5.95	1.004
$y_{5,t}$	6.65	0.80	6.23	1.01
$y_{10,t}$	7.13	0.54	6.55	1.02

The table presents a comparison of model-derived moments with observed data moments for zero-coupon yield curve (ZCYC) yields across various maturities: one year ($y_{1,t}$), two years ($y_{2,t}$), five years ($y_{5,t}$), and ten years ($y_{10,t}$). This analysis focuses on the post-IT regime, specifically spanning the period from July 2015 to March 2021.

function (ACF) in Table 3 successfully reproduces the persistence.

Much of the observed term premium is driven by the inflation risk premium. The estimated average inflation risk premium through the counterfactual analysis is shown in Table 5. In the first counterfactual scenario, the term premium is estimated under the assumption that the representative agent internalizes a Central Bank policy predominantly oriented toward active stabilization of inflation while forming long-term expectations regarding inflation. The estimated smoothed probabilities derived from the benchmark model are employed for the calculation of the counterfactual term premium. Furthermore, all other parameters, with the exception of the policy coefficient, are maintained at their posterior mean estimates from the benchmark scenario. This calibration yields a reduction in the average term premium of approximately 160 basis points, thereby implying an inflation risk premium of a comparable magnitude.

Table 5: **The counterfactual asset pricing moment**

Variable	Data	Case I	Case II
$y_{1,t}$	6.56	6.85	6.77
$y_{2,t}$	6.8	6.87	6.84
$y_{5,t}$	7.26	6.91	7.02
$y_{10,t}$	7.63	6.97	7.17
r_m	13.6	14.2	14
$\sigma(rm)$	0.26	0.23	0.24
$Termpremium$	198bps	34bps	133bps

The table presents the estimation of the Inflation Risk Premium. This premium is assessed under two counterfactual scenarios: Case I) active inflation stabilization with $\xi_p = 1.25$, while keeping other parameters fixed at their posterior means; Case II) a 50% reduction in the innovations related to inflation target assumptions $\alpha(-)$, with all other parameters held constant at their posterior means.

In the second scenario, the improvements in the term premium components are quantified by reducing the covariance term by 50% in the inflation target latent equation. All other parameters, including the policy coefficients, are held constant at the estimated posterior from the benchmark case. This analysis simulates a situation where alternative policy tools are favoured over active inflation stabilization to keep inflation close to the established target. Under these conditions, the improvements in the term premium are observed to be between 50 and 60 basis points.

4 Conclusion

The paper explores the underlying mechanism behind persistent term premium in the Indian ten-year government bond market, particularly in the current Inflation-Targeting regime in the context of unanchored long-run inflation expectations. Utilizing a Q-learning algorithm featuring a New Keynesian DSGE, the paper empirically demonstrates that the downward revision in long-term inflation expectations among bond market participants following the implementation of an inflation targeting framework is primarily due to the adoption of the lower inflation target of 4% by the Central Bank.

However, bond market participants continue to display unanchored expectations around this target. Additionally, these participants exhibit backward-looking behaviour and continue to index to realized inflation while deriving their forward-looking inflation expectations. The transmission of short-term forecast errors to long-term revisions of inflation targets remains considerable. This uncertainty largely arises from repeated breaches of the upper rational bounds with short-term inflation expectations that participants typically set based on the Central Bank's inflation target. These recurrent breaches tend to be countercyclical with respect to real consumption growth, leading individual bond market participants to assign a substantial probability to the possibility of significant future upward revisions of inflation target assumptions by other bond market participants.

As a result, forward-looking expectations with large deviations from 4% inflation-target are consistently integrated into the pricing mechanisms used by bond market participants, even within the current Inflation-targeting regime. This integration is a key factor contributing to the persistent levels of term premiums observed in the bond market.

Appendices

Appendix of the Q-learning algorithm

A.1 Derivation of Γ

Substituting for marginal cost in the log-linearized aggregate supply curve:

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \mu_t + E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \Big[\tau_p \widehat{mc}_t + (1-\gamma)\beta(\hat{\pi}_{T+1} - \gamma_p \hat{\pi}_T) \Big]$$

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \tilde{\mu}_t + \tilde{\kappa}\epsilon_t + E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \Big[\epsilon_{T+1} - (\hat{\pi}_{T+1} - \bar{\pi}_T) + \gamma_p(\hat{\pi}_T - \bar{\pi}_T) \Big]$$
$$+ E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \Big[(1-\gamma)\tilde{\beta}(\hat{\pi}_{T+1} - \gamma_p \hat{\pi_T}) \Big]$$

where $\tilde{\kappa} = (1 + \frac{\tau_p}{1+\Gamma_x\phi})^{-1} \frac{\tau_p}{\Gamma_x\phi}$ and $\tilde{\beta} = \frac{\beta}{1+(\Gamma_x\phi)^{-1}}$

Which can be further expanded as:

$$\tilde{\kappa}\epsilon_t + E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \Big[\gamma\beta\tilde{\kappa}\epsilon_{T+1} - (1-\gamma\beta\gamma_p)(\gamma\beta\tilde{\kappa} - (1-\gamma)\tilde{\beta})\hat{\pi}_{T+1} \Big]$$
$$+ E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \Big[\gamma\beta\tilde{\kappa}(1-\gamma)\bar{\pi}_t \Big] + \tilde{\mu} = (1 + (1-\gamma)\tilde{\beta} - \gamma\beta\tilde{\kappa})\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}$$

The rational expectation equilibrium can be simplified as:

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \tilde{\mu}_t + \tilde{\kappa} E_t \sum_{T=t}^{\infty} \tilde{\beta}^{T-t} \epsilon_T$$
$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \tilde{\mu}_t + \bar{\omega}\epsilon_t$$

where, $\bar{\omega} = \frac{\tilde{\kappa}}{1-\tilde{\beta}\rho}$

Substituting for $\tilde{\kappa}$ and $\tilde{\beta}$ we have expression for $\bar{\omega}$ as in equation (2.8). Substituting the discounted value for forecasted inflation, which can be expanded as:

$$E_t\sum_{T=t}^{\infty}(\gamma\beta)^{T-t}\hat{\pi}_{T+1} = (\frac{1}{1-\gamma\beta} - \frac{\gamma_p}{1-\gamma\beta\gamma_p})\bar{\pi}_t + \frac{\gamma_p}{1-\gamma\beta\gamma_p}\hat{\pi}_t + \frac{\bar{\omega}\rho}{(1-\gamma\beta\rho)(1-\gamma\beta\gamma_p)}\epsilon_t$$

into the aggregate supply equation

$$\begin{aligned}\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \tilde{\mu}_t + \left[\tilde{\kappa} + \frac{\gamma\beta\tilde{\kappa}\rho}{1-\gamma\beta\rho} - \frac{(1-\gamma\beta\gamma_p)(\gamma\beta\tilde{\kappa}-(1-\gamma)\tilde{\beta})}{(1-\gamma\beta\gamma_p)(1-\gamma\beta\rho)}\bar{\omega}\rho\right]\epsilon_T \\ + \left[\gamma\beta\tilde{\kappa}\frac{1-\gamma_p}{1-\gamma\beta} + ((1-\gamma)\tilde{\beta} - \gamma\beta\tilde{\kappa})(\frac{1-\gamma\beta\gamma_p}{1-\gamma\beta} - \frac{1-(\gamma\beta\gamma_p)\gamma_p}{1-\gamma\beta\gamma_p})\right]\bar{\pi}_t\end{aligned}$$

Using the fact that,

$$\tilde{\kappa} + \frac{\gamma\beta\tilde{\kappa}\rho}{1-\gamma\beta\rho} - \frac{(1-\gamma\beta\gamma_p)\Big(\gamma\beta\tilde{\kappa}-(1-\gamma)\tilde{\beta}\Big)}{(1-\gamma\beta\rho)(1-\gamma\beta\gamma_p)}\bar{\omega}\rho = \bar{\omega}$$

allows the following expression for the supply curve :

$$\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \tilde{\mu}_t + \bar{\omega}\epsilon_t + \left[\gamma\beta\tilde{\kappa}\frac{1-\gamma_p}{1-\gamma\beta} + \left(\frac{1-\gamma\beta\gamma_p}{1-\gamma\beta} - \gamma_p\right)\right]\bar{\pi}_t$$

The supply curve can be shown as:

$$\begin{aligned}\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \tilde{\mu}_t + \bar{\omega}\epsilon_t + \frac{1}{\tau_p+\Gamma_x\phi}\left[\gamma\beta\gamma_p\left(\frac{1-\gamma_p}{1-\beta\gamma} - \frac{1-\gamma\beta\gamma_p}{1-\gamma\beta}\right)\right]\bar{\pi}_t \\ + \frac{1}{\tau_p+\Gamma_x\phi}\left[(1-\gamma)\beta\Gamma_x\phi\left(\frac{1-\gamma\beta\gamma_p}{1-\gamma\beta} - \gamma_p\right) + \gamma\beta\tau_p\gamma_p\right]\bar{\pi}_t\end{aligned}$$

or,

$$\hat{\pi}_t = \gamma\hat{\pi}_{t-1} + (1-\gamma)\Gamma\bar{\pi}_t + \rho\bar{\omega}\epsilon_{t-1} + \bar{\omega}\epsilon_t$$

where

$$\Gamma = \frac{1}{1+\phi^{-1}\tau_p\Gamma_x^{-1}}\frac{(1-\gamma)\beta}{1-\gamma\beta}$$

Putting Φ =1 we get,

$$\Gamma = \frac{1}{1+\tau_p\Gamma_x^{-1}}\frac{(1-\gamma)\beta}{1-\gamma\beta}$$

A.2 Marginalised Particle Filter

The model summary can be expressed as:

$$\begin{aligned}
\hat{\pi}_t &= (1-\gamma)\Gamma\bar{\pi}_t + \gamma\hat{\pi}_{t-1} + s_t + \mu_t \\
\bar{\pi}_t &= \bar{\pi}_{t-1} + g_{t-1}^{-1} e_{t-1} \\
e_t &= (1-\gamma)(\Gamma - 1)\bar{\pi}_t + \mu_t + \epsilon_t
\end{aligned}$$

$$g_t = \begin{cases} g_{t-1} + 1, & \text{if } |\ \hat{E}_{t-1}\hat{\pi}_t - E_{t-1}\hat{\pi}_t\ | \leq \Theta * \sqrt{MSE} \rightarrow \textbf{Decreasing gain} \\ \bar{g}^{-1}, & \text{otherwise} \rightarrow \textbf{Constant Gain} \end{cases} \tag{23}$$

$$s_t = \rho_s s_{t-1} + \epsilon_t$$

The model can be rewritten as:

$$\begin{aligned}
g_t &= f_g(\bar{\pi}_{t-1}, g_{t-1}) \\
\bar{\pi}_t &= f_\pi(\bar{\pi}_{t-1}, g_{t-1}) + f_g(\bar{\pi}_{t-1}, g_{t-1})^{-1}\eta_{t-1} \\
\eta_t &= \mu_t + \epsilon_t \\
s_t &= \rho_s s_{t-1} + \epsilon_t \\
\hat{\pi}_t &= (1-\gamma_p)\Gamma f_\pi(\bar{\pi}_{t-1}, g_{t-1}) + (1-\gamma_p)\Gamma f_g(\bar{\pi}_{t-1}, g_{t-1})^{-1}\eta_{t-1} \\
&\quad + \gamma_p\hat{\pi}_{t-1} + \rho s_{t-1} + \epsilon_t + \mu_t
\end{aligned}$$

$$\text{where,} \;\; f_g(\bar{\pi}_{t-1}, g_{t-1}) = I(\bar{\pi}_{t-1}) * (k_{t-1} + 1) + (1 - I(\bar{\pi}_{t-1})) * g^{-1}$$

and, $I(\bar{\pi}_t)$ is the indicator function with the gain function as defined above.

$$f_\pi(\bar{\pi}_{t-1}, g_{t-1}) = \left[1 - (1-\Gamma)(1-\gamma_p) f_g(\bar{\pi}_{t-1}, g_{t-1})^{-1}\right]\bar{\pi}_{t-1}$$

The above system can be written by separating linear(represented by $\mathcal{E}_t$) and non linear states as shown below:

$$\mathcal{E}_t = f_{\mathcal{E}_t}(\bar{\pi}_{t-1}, g_{t-1}) + A_{\mathcal{E}_t}(\bar{\pi}_{t-1}, g_{t-1})\mathcal{E}_t + B_{\mathcal{E}_t}\begin{bmatrix}\epsilon_t \\ \mu_t\end{bmatrix}$$

$$B_{\mathcal{E}} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

For the nonlinear variable, the system can be expressed as:

$$\begin{aligned} g_t &= f_g(\bar{\pi}_{t-1}, g_{t-1}) \\ \text{and, } \bar{\pi}_t &= f_{\bar{\pi}}(\bar{\pi}_{t-1}, g_{t-1}) + A_{\bar{\pi}}(\bar{\pi}_{t-1}, g_{t-1})^{-1}\mathcal{E}_t \end{aligned}$$

$$\text{where, } A_{\bar{\pi}}(\bar{\pi}_{t-1}, g_{t-1}) = \begin{bmatrix} f_g(\bar{\pi}_{t-1}, g_{t-1}) \\ 0_{2\times 1} \end{bmatrix}^{-1}$$

The gain function does not depend on the linear state variables, and the system can be summarised as:

$$g_t = f_g(\bar{\pi}_{t-1}, g_{t-1})$$

$$\begin{bmatrix} \bar{\pi}_t \\ \mathcal{E}_t \end{bmatrix} = f(\bar{\pi}_{t-1}, g_{t-1}) + A(\bar{\pi}_{t-1}, g_{t-1})\mathcal{E}_{t-1} + \begin{bmatrix} 0 \\ S_{\mathcal{E}} \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}$$

$$f(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} f_\pi(\bar{\pi}_{t-1}, k_{t-1}) \\ f_{\mathcal{E}}(\bar{\pi}_{t-1}, k_{t-1}) \end{bmatrix}$$

$$A(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} A_\pi(\bar{\pi}_{t-1}, k_{t-1}) \\ A_{\mathcal{E}}(\bar{\pi}_{t-1}, k_{t-1}) \end{bmatrix}$$

$$and, \Sigma = \left[E\left(\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}' \right) \right]$$

The quarterly observation variables are linked in the following manner:

$$Y_t = \begin{bmatrix} \pi_t \\ E_t^{SPF}\pi_{t+1} \\ E_t^{SPF}\pi_{t+2} \\ E_t^{SPF}\pi_{t+3} \end{bmatrix} = i_{0|t} + i_{\bar{\pi},t}\bar{\pi}_t + I_t'\mathcal{E}_t + Q_t^{1/2}e_t^o \tag{24}$$

The measurement errors for all the observed variables are represented by vector o_t. The matrices I_t and Q_t are time-varying to account for missing observations.

The marginal particle filter follows the algorithm of Schön et al. 2005 and the importance sampling is done following Andrieu et al. 2010 and Doucet et al. 2001.The following distribution is targeted:

$$P\Big(\mathcal{E}_t, \Big[\bar{\pi}_t, k_t\Big]|Y_t\Big) = P\Big(\mathcal{E}_t|\Big[\bar{\pi}_t, k_t\Big], Y_t\Big) \times P\Big(\Big[\bar{\pi}_t, k_t\Big]|Y_t\Big) \tag{25}$$

The algorithm is as follows:

- Initially, choose $\bar{\pi}_{1|0}^{(i)}$, $k_{1|0}^{(i)}$ from the distribution, and the $\mathcal{E}_{1|0}^{(i)}$, $\mathcal{P}_{1|0}^{(i)} = [\mathcal{E}_0, P_{1|0}]$ where $P_{1|0}$ is the initial precision matrix in the linear Kalman filter.

- For each time t= 1....T, compute $\Omega_t = I_t^{'} P_{t|t-1} I_t + Q_t$ and its inverse. For i= 1,.....,N the importance weights are estimated for the sampling purpose $q_t^{(i)} = P(y_t|\bar{\pi}_{t|t-1}^{(i)}, \mathcal{E}_{t|t-1}^{(i)})$

- where, $P(y_t|\bar{\pi}_{t|t-1}^{(i)}, \mathcal{E}_{t|t-1}^{(i)}) = N(i_{0|t} + i_{\bar{\pi}_t}\bar{\pi}_{t|t-1}^{(i)} + I_t^{'}\mathcal{E}_{t|t-1}^{i}, I_t^{'} P_{t|t-1} I_t + Q_t)$

- and,$q_t^i = w_{t-1}^i \times |\Omega|^{-1/2} \times$

  $\exp\Big[-\frac{1}{2}(y_t - i_{0|t} - i_{\bar{\pi}_t}\bar{\pi}_{t|t-1}^{(i)} - I_t^{'}\mathcal{E}_{t|t-1}^{i})^{'} \times \Omega_t^{-1} \times (y_t - i_{0|t} - i_{\bar{\pi}_t}\bar{\pi}_{t|t-1}^{(i)} - I_t^{'}\mathcal{E}_{t|t-1}^{i})\Big]$

  where w_{t-1}^i is the particle weight estimated in the previous period.

- The effective sample size is computed as: $ESS = \frac{1}{\sum {w_t^j}^2}$

- When the effective sample size falls below $0.75 \times N$, the resampling is done to remove particles with low weight in the following manner:

$$P\left(\left[\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}\right] = \left[\bar{\pi}^{(i)}_{t|t-1}, k^{(i)}_{t|t-1}\right]\right) = \frac{q^{(j)}_t}{\sum q^{(j)}_t}$$

- The systematic resampling is done following Doucet et al. 2001. The systematic resampling results in discrete distribution with particles $\left[\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}\right]^N_{i=1}$ and corresponding weights $w_t(i) = 1/N$ for i= 1.......N. If the resampling is not done, the weights are expressed as $w^i_{t-1} = \frac{q^{(j)}_t}{\sum q^{(j)}_t}$

- Linear measurement equation: for i =1.....N, evaluate:

$$\mathcal{E}^{(i)}_{t|t} = \mathcal{E}^{(i)}_{t|t-1} + K_t(y_t - i_{0|t} - i_{\bar{\pi}_t}\bar{\pi}^{(i)}_{t|t-1} - I^{'}_t\mathcal{E}^{i}_{t|t-1})$$

$$K_t = P_{t|t-1}H_t\Omega^{-1}_t$$

$$P_{t|t} = P_{t|t-1} - K_tH^{'}_tP_{t|t-1}$$

- Particle filter prediction. For $i = 1.....N$ compute $k^{(i)}_{t+1|t} = f_k(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t})$ and then draw $\pi^{(i)}_{t+1|t}$ from distribution:

$$P\left(\bar{\pi}_{t+1}|Y_t, \bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}\right) = N\left(f_\pi(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) + f_k([\bar{\pi}^{(i)}_{t|t}], [k^{(i)}_{t|t}])^{-1}z^i_{t|t},\right.$$
$$\left. f_k([\bar{\pi}^{(i)}_{t|t}], [k^{(i)}_{t|t}])^{-1}[z^{(i)}_{t|t}]^{-2}P^{[\eta,\eta]}_{t|t}\right)$$

where the notation $P^{[x,z]}_{t|t} = P_{t|t}(x,z)$

- Linear Model Predictions are done in the following manner:

$$\tilde{\mathcal{E}}^{(i)}_{t|t} = \mathcal{E}^{(i)}_{t|t} + \tilde{K}^{(i)}_t \Big(\bar{\pi}^{(i)}_{t+1|t} - f_{\bar{\pi}}(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) - f_k(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t})^{-1} z^{(i)}_{t|t} \Big)$$

$$\mathcal{E}^{(i)}_{t+1|t} = f_{\mathcal{E}}(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) + A_{\mathcal{E}}(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) \tilde{\mathcal{E}}^{(i)}_{t|t}$$

$$P_{t+1|t} = R_{\mathcal{E}} + \tilde{P}_{t|t}; R_{\mathcal{E}} = B_{\mathcal{E}} \Sigma B^{'}_{\mathcal{E}}$$

$$\tilde{K}^i_t = P_{t|t} A^{'}_{\pi} \Big(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t} \Big) \Big(A_{\bar{\pi}}(\pi^i_{t|t}, k^i_{t|t}) P_{t|t} A^{'}_{\bar{\pi}}(\pi^i_{t|t}, k^i_{t|t}) \Big)^{-1}$$

$$f_k(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) = \begin{bmatrix} 1 \\ P^{[\eta,s]}_{t|t} / P^{[\eta,\eta]}_{t|t} \\ P^{[\eta,\pi]}_{t|t} / P^{[\eta,\eta]}_{t|t} \end{bmatrix}$$

$$A_{linear}(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) \tilde{K}^i_t = \begin{bmatrix} 0 \\ \frac{(P^{\eta,s}_{t|t})}{P^{\eta,\eta}_{t|t}} \rho \\ \frac{(P^{\eta,s}_{t|t})}{P^{\eta,\eta}_{t|t}} \rho + \frac{(P^{\eta,\pi}_{t|t})}{P^{\eta,\eta}_{t|t}} \gamma_p + f_k(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t}) \} \gamma_p + f_k(\bar{\pi}^{(i)}_{t|t}, k^{(i)}_{t|t})^{-1} (1 - \gamma_p) \Gamma \end{bmatrix}$$

$$\tilde{P}_{t|t} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \tau_1 & \tau_1 + \tau_2 \\ 0 & \tau_1 + \tau_2 & \tau_1 + 2\tau_2 + \Big(P^{\pi,\pi}_{t|t} - \frac{(P^{\eta,\pi}_{t|t})^2}{P^{\eta,\eta}_{t|t}} \gamma_p^2 \Big) \end{bmatrix}$$

$$\text{where, } \tau_1 = \Big(P^{s,s}_{t|t} - \frac{(P^{\eta,s}_{t|t})^2}{P^{\eta,\eta}_{t|t}} \rho^2 \Big)$$

$$\tau_1 = \Big(P^{s,\pi}_{t|t} - \frac{(P^{\eta,s}_{t|t})^2}{P^{\eta,\eta}_{t|t}} \rho^2 \Big) and, \tau_2 = \Big(P^{s,\pi}_{t|t} - \frac{(P^{\eta,s}_{t|t})(P^{\eta,\pi}_{t|t})}{P^{\eta,\eta}_{t|t}} \rho \gamma_p \Big)$$

Finally, the log-likelihood is estimated by:

$$L = \sum_{t=1}^{T} logP(y_t|Y_{t-1})$$

$$\begin{aligned} \text{where } P(y_t|Y_t) &= P(y_t|\mathcal{E}_t, [\bar{\pi}_t, k_t])p(\mathcal{E}_t, [\bar{\pi}_t, k_t]|Y_{t-1}) \\ &= P(y_t|\mathcal{E}_t, [\bar{\pi}_t, k_t])P(\mathcal{E}_t|\left[\bar{\pi}_t, k_t\right], Y_{t-1})P(\left[\bar{\pi}_t, k_t\right]|Y_{t-1}) \end{aligned}$$

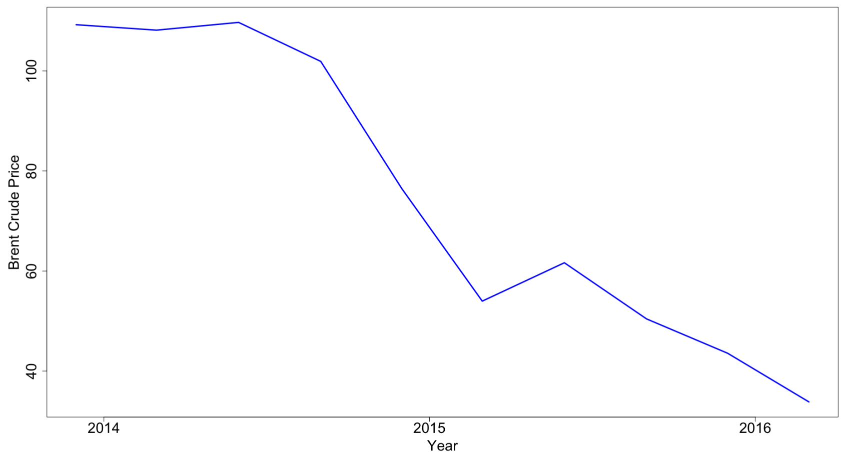
The likelihood can be summarised as: $L \approx \sum_{t=1}^{T} log(\sum_{i=1}^{i=N} q_t^{(i)})$

A.3 Source of Data used in training the Q-learning model

Inflation is measured using the headline Consumer Price Index (CPI). The headline CPI data is available at a quarterly frequency from the OECD/CEIC database, starting in September 1958. To align with the frequency available in the SPF, twelve-month rolling inflation is calculated each quarter. The inflation forecasts for one, two, and three quarters ahead are sourced from SPF surveys conducted by the Central Bank. The SPF data series are available quarterly from September 2007. Since the fiscal year 2014, SPF forecasts are released twice each quarter. For the quarters beginning in 2014, the analysis uses the average of both forecasts.

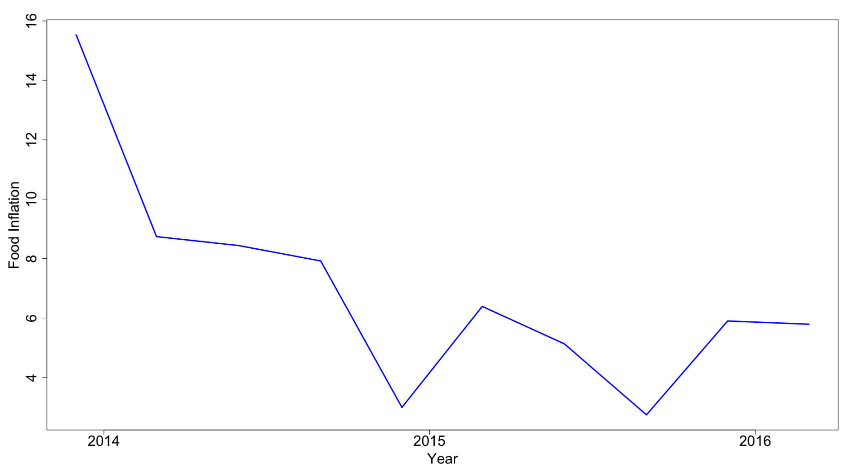
A.4 Crude Oil Price and Food Inflation Plots

Figure A.4.1: **The Brent Crude Oil Price**



The figure shows the Spot Price of European Brent Crude Oil Prices at the quarterly frequency, sourced from CEIC (U.S. Energy Information Administration) for the period Q3:2013 to Q3:2016.

Figure A.4.2: **Food Inflation India**



The figure shows India's food inflation at the quarterly frequency, sourced from the Ministry of Statistics and Program Implementation for the period Q3:2013 to Q3:2016.

Appendix of the Asset Pricing Model

A.5 Approximate analytical solutions

The representative agent has an Epstein-Zin type recursive preference like in Epstein et al. (2013) and Bansal and Yaron (2004) and maximizes its lifetime utility with consumption stream:

$$U_t = max_{C_t}((1-\delta)C_t^{\frac{1-\gamma}{\kappa}} + \delta(E_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\kappa}})^{\frac{\kappa}{1-\gamma}} \tag{26}$$

where the log SDF is expressed as $q_{t+1} = \kappa log\delta - \frac{\kappa}{\omega}\Delta c_{t+1} + (\kappa - 1)re_{a,t+1}$ $re_{a,t}$ is the log return with the portfolio in which the total wealth of the agent is invested. Such portfolios can be compared with CAPM-like efficient portfolios. This portfolio comprises i) observable components, like aggregate equity and bonds, and ii) unobservable components, like human potential, which drives the economy to a higher growth phase, resulting in higher wealth creation. Consumption can be seen as the dividend associated with this portfolio. The budget constraint is defined as $W_{t+1} = (W_t - C_t)R_{a,t+1}$. The F.O.C. gives the Euler equations which can be defined for any i^{th} asset as:

$$E_t\left[exp(q_{t+1} + re_{i,t+1})\right] = 1, i \in a, m : Real Assets$$
$$logE_t\left[exp(pr^{\$}_{n-1,t+1} + q_{t+1} - \pi_{t+1})\right] = pr^{\$}_{n,t} : Nominal Assets$$

The returns with each real asset are approximated following Campbell and Shiller 1991:

$$re_{a,t+1} = \upsilon_{0,c} + \upsilon_{1,c}pd_{c,t+1} - pd_{c,t} + gr_{c,t+1}$$
$$re_{m,t+1} = \upsilon_{0,m} + \upsilon_{1,m}pd_{m,t+1} - pd_{m,t} + gr_{m,t+1}$$

where $pd_{c,t}$ $pd_{m,t}$ are the log price to dividend with consumption and dividend stream, respectively, and gr are the respective growths in the consumption and dividend stream. The above Euler equations are expanded using the law of iterated expectations following Bansal and Zhou 2002.

$$E\left[E\left[exp(q_{t+1}+re_{m,t+1})|Z_{t+1}\right]|Z_t\right]=1$$
$$\Sigma_{k=1}^{2}\Delta_{ik}E\left(exp(q_{t+1}+re_{m,t+1})|St_{t+1}=k,Y_t\right)=1$$

Where Z_t is the information available at time t regarding the states and regime, $St_t \in k$ and the state variable Y_t.

Now, using the approximation exp(A) -1 $\approx$ A and following the assumption of log-normality, we get the expanded Euler equation as:

$$\Sigma_{k=1}^{2}\Delta_{ij}\left(E\left[(q_{t+1}+re_{m,t+1})|St_{t+1}=k,Y_t\right]+\frac{1}{2}V\left[(q_{t+1}+re_{m,t+1})|St_{t+1}=k,Y_t\right]\right)=0$$

A.6 The Transition Probability

The shocks to the latent process follow a two-state Markov chain $St_t \in (procyclical, countercyclical)$. The Markov transition probability matrix is given below where $\Sigma_{j=pro}^{j=coun}\Delta_{ij}=1$ $\forall$ i $\in$ (pro,coun):

$$\Delta=\begin{bmatrix} p_{pro,pro} & p_{pro,coun} \\ p_{coun,pro} & p_{coun,coun} \end{bmatrix}$$

For ease of notation purpose, $St_t \in (procyclical, countercyclical)$ is represented as $St_t \in (1,2)$ in the subsequent sections.

A.7 The Linearization parameters

The linearization parameters are determined numerically until reaching a fixed point for pd_i for $i \in (c,m)$:

$$\bar{pd}_i=\sum_{k=1}^{k=2}\bar{p}_j C_{0,i}(k)$$
$$v_{0,i}=\frac{exp(\bar{pd}_i)}{1+exp(\bar{pd}_i)}$$

$$\upsilon_{1,i} = log(1 + exp(\bar{pd_i})) - \upsilon_{1,i}\bar{pd_i}$$

where, $\bar{p}_j = \sum_{k=1}^{k=2} \bar{p}_i \Delta_{ij}$

A.8 Real Consumption streams

The log price-consumption ratio is conjectured to be exponentially affine as:

$$pd_{c,t} = C_0(St_t) + C_1(St_t)Y_t$$

The real return with the consumption process can be eventually shown as:

$$\begin{aligned} re_{c,t+1} &= \upsilon_0 + \mu_c + \upsilon_1 C_0(St_{t+1}) - C_0(St_t) \\ &+ (a_1 + \upsilon_1 C_1(St_{t+1})\Psi_1(St_{t+1}) - C_1(St_t))Y_t \\ &+ \upsilon_1 C_1(St_{t+1})\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1} + a_1 \Sigma \eta_{t+1} \end{aligned}$$

The solution for the C is derived by substituting $re_{c,t}$ in the expanded Euler equation $E\left[(q_{t+1} + re_{c,t+1})\right] + \frac{1}{2} V\left[(q_{t+1} + re_{c,t+1})\right] = 0$:

$$\begin{bmatrix} C_1(1)' \\ C_1(2)' \end{bmatrix} = \begin{bmatrix} I - p11\upsilon_1\Psi_1 & -p12\upsilon_1\Psi_1 \\ -p21\upsilon_1\Psi_1 & I - p22\upsilon_1\Psi_1 \end{bmatrix}^{-1} (1 - \frac{1}{\omega}) \begin{bmatrix} a_1' \\ a_1' \end{bmatrix}$$

$$\begin{bmatrix} C_0(1) \\ C_0(2) \end{bmatrix} = (I - \upsilon_1\Delta)^{-1}\Delta \begin{bmatrix} log\delta + \upsilon_0 + (1 - \frac{1}{\omega})\mu_c + \frac{\kappa}{2}(1 - \frac{1}{\omega})^2 a_1 \Sigma \Sigma' a_1' + \frac{\kappa}{2}\Upsilon(1)\Upsilon(1)' \\ log\delta + \upsilon_0 + (1 - \frac{1}{\omega})\mu_c + \frac{\kappa}{2}(1 - \frac{1}{\omega})^2 a_1 \Sigma \Sigma' a_1' + \frac{\kappa}{2}\Upsilon(2)\Upsilon(2)' \end{bmatrix}$$

where, $\Upsilon(S_t) = \upsilon_1 C_1(St_{t+1})\Psi_2(S_{t+1})\Sigma y(S_{t+1})$. The log stochastic discount factor is expressed as:

$$\begin{aligned} q_{t+1} = \kappa log\delta + (\kappa - 1)(\upsilon_0 + \upsilon_1 C_0(St_{t+1}) - C_0(St_t)) - \gamma\mu_c - \frac{1}{\omega} a_1 Y_t \\ + (\kappa - 1)((1 - \frac{1}{\omega})a_1 + \upsilon_1 C_1(St_{t+1})\Psi_1(St_{t+1}) - C_1(St_t))Y_t - \gamma a_1 \Sigma \eta_{t+1} \end{aligned}$$

$$(\kappa - 1)\upsilon_1 C_1(St_{t+1})\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1}$$

A.9 Real Dividend Process

Similarly, the log price-dividend with aggregate equity market is conjectured as:

$$pd_{m,t} = C_{0,m}(St_t) + C_{1,m}(St_t)Y_t$$

and the return with the dividend stream is further expressed as:

$$\begin{aligned} re_{m,t+1} &= \upsilon_{0,m} + \mu_d + \upsilon_{1,m}C_{0,m}(St_{t+1}) - C_{0,m}(St_t) \\ &+ (\phi a_1 + \upsilon_{1,m}C_{1,m}(St_{t+1})\Psi_1(St_{t+1}) - C_{1,m}(St_t))Y_t \\ &+ \upsilon_{1,m}C_{1,m}(St_{t+1})\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1} + a_2\Sigma\eta_{t+1} \end{aligned}$$

where, $C_{m,s}$ have the following solutions:

$$\begin{bmatrix} C_{1,m}(1)^{'} \\ C_{1,m}(2)^{'} \end{bmatrix} = \begin{bmatrix} I - p11\upsilon_{1,m}\Psi_1 & -p12\upsilon_{1,m}\Psi_1 \\ -p21\upsilon_{1,m}\Psi_1 & I - p22\upsilon_{1,m}\Psi_1 \end{bmatrix}^{-1} (\phi - \frac{1}{\omega}) \begin{bmatrix} a_1^{'} \\ a_1^{'} \end{bmatrix}$$

$$\begin{bmatrix} C_{0,m}(1) \\ C_{0,m}(2) \end{bmatrix} = (I - \upsilon_{1,m}\Delta)^{-1} \big(\Delta \big[\begin{bmatrix} (\kappa-1)\upsilon_1 C_0(1) + \frac{1}{2}\Upsilon_m(1)\Upsilon_m(1)^{'} \\ (\kappa-1)\upsilon_1 C_0(2) + \frac{1}{2}\Upsilon_m(2)\Upsilon_m(2)^{'} \end{bmatrix} \big] + \big[\Lambda_m(1) \quad \Lambda_m(2) \big]^{'} \big)$$

$$\begin{aligned} where, \Lambda_m(St_t) &= \kappa log\delta + (\kappa-1)(\upsilon_0 - C_0(St_t)) - \gamma\mu_c \\ &+ \upsilon_{0,m} + \mu_d + \frac{1}{2}(\gamma^2 a_1\Sigma\Sigma^{'} a_1^{'} + a_2\Sigma\Sigma^{'} a_2^{'}) \\ and, \Upsilon_m(St_t) &= \big((\kappa-1)\upsilon_1 C_1(St_{t+1}) + \upsilon_{1,m}C_{1,m}(St_{t+1})\big)\Psi_2(St_t)\Sigma y(St_t) \end{aligned}$$

A.10 Linerization parameters

The linearization parameters are solved in the following manner:

$$\overline{pd}_i = \sum_{j=1}^{2} \overline{p}_j C_{0,i}(j)$$

$$\upsilon_{1,i} = \frac{exp(\overline{pd}_i)}{1 + exp(\overline{pd}_i)}$$

$$\upsilon_{0,i} = log(1+exp(\overline{pd}_i)) - \upsilon_{1,i}\overline{pd}_i$$
$$\bar{p}_j = \sum_{i\in 2} \bar{p}_i \Delta_{ij}$$

The solution is solved numerically to achieve a fixed point of $\bar{pd}_i$ for i $\in(c,m)$.

A.11 Inflation Process

The monetary policy rule is defined as:

$$i_t = \tau_0(St_t) + \tau_c(St_t)y_{c,t} + \tau_\pi(St_t)(\pi_t - \Theta_0(St_t) - y_{\pi,t}) + y_{pi,t} + y_{i,t}$$

The inflation process is conjectured to be linear in the form:

$$\pi_t = \Theta_0(St_t) + \underbrace{(\Theta_{1,c}(St_t) \quad , \Theta_{1,\pi}(St_t) \quad , \Theta_{1,i}(St_t))}_{\Theta_1} Y_t$$

Combining the above two equations, the monetary policy rule is expanded as:

$$i_t = \tau_0(St_t) + \Big[\tau_c(St_t) + \tau_\pi(St_t)\Theta_{1,c}(St_t) \quad , 1 - \tau_\pi(St_t) +$$
$$\tau_\pi(St_t)\Theta_{1,\pi}(St_t) \quad , 1 + \tau_\pi(St_t)\Theta_{1,i}(St_t)\Big] Y_t$$

The asset pricing solution gives:

$$i_t = -E_t(q_{t+1} - \pi_{t+1}) - \frac{1}{2}Var_t(q_{t+1} - \pi_{t+1})$$

Combining the asset pricing solution and the expanded monetary policy rule, we get:

$$\begin{bmatrix} \Theta_{1,c}(1) \\ \Theta_{1,c}(2) \end{bmatrix} = \left(\begin{bmatrix} \tau_\pi & 0 \\ 0 & \tau_\pi \end{bmatrix} - \Delta \begin{bmatrix} \varphi_c & 0 \\ 0 & \varphi_c \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{1}{\psi} - \tau_c \\ \frac{1}{\psi} - \tau_c \end{bmatrix}$$

$$\begin{bmatrix}\Theta_{1,\pi}(1)\\ \Theta_{1,\pi}(2)\end{bmatrix} = \left(\begin{bmatrix}\tau_\pi & 0\\ 0 & \tau_\pi\end{bmatrix} - \Delta\begin{bmatrix}\varphi_\pi & 0\\ 0 & \varphi_\pi\end{bmatrix}\right)^{-1}\begin{bmatrix}\tau_\pi - 1\\ \tau_\pi - 1\end{bmatrix}$$

$$\begin{bmatrix}\Theta_{1,i}(1)\\ \Theta_{1,i}(2)\end{bmatrix} = \left(\begin{bmatrix}\tau_\pi & 0\\ 0 & \tau_\pi\end{bmatrix} - \Delta\begin{bmatrix}\varphi_i & 0\\ 0 & \varphi_i\end{bmatrix}\right)^{-1}\begin{bmatrix}1\\ 1\end{bmatrix}$$

and the constant:

$$\begin{bmatrix}\Theta_0(1)\\ \Theta_0(2)\end{bmatrix} = \Delta^{-1}\begin{bmatrix}\Psi_\pi(1) & \Psi_\pi(2)\end{bmatrix} + \begin{bmatrix}\tau_\pi(1) & \tau_\pi(2)\end{bmatrix}$$

$$\begin{aligned}\text{where, } \tau_\pi(St_t) &= (\kappa-1)C_0(St_t)\upsilon_1 + \frac{1}{2}[((\kappa-1)\upsilon_1 C_1(St_t) - \Theta_1(St_t))\Psi_2(St_t)\Sigma y(St_t)]\\ &\quad [((\kappa-1)\upsilon_1 C_1(St_t) - \Theta_1(St_t))\Psi_2(St_t)\Sigma y(St_t)]'\\ \Psi_\pi(S_t) &= \tau_0(St_t) + \left(\kappa log\delta + (\kappa-1)\upsilon_0 - \gamma\mu_c + \frac{\gamma^2}{2}a_1\Sigma\Sigma' a_1'\right) - (\kappa-1)C_0(St_t)\end{aligned}$$

A.12 Nominal bond pricing solutions

The nominal log stochastic discount factor is:

$$\begin{aligned}q_{t+1} - \pi_{t+1} &= q^{\$}_{t+1}\\ &= \kappa log\delta + (\kappa-1)(\upsilon_0 + \upsilon_1 C_0(St_{t+1}) - C_0(St_t)\\ &\quad - \gamma\mu_c - \Theta_0(St_{t+1})\\ &\quad - \big(\frac{1}{\omega}a_1 + \Theta_1(St_{t+1})\Psi_1(St_{t+1})\big)Y_t\\ &\quad + (\kappa-1)\big((1-\frac{1}{\omega})a_1 + \upsilon_1 C_1(St_{t+1})\Psi_1(St_{t+1}) - C_1(St_t)\big)Y_t\\ &\quad -\gamma a_1\Sigma\eta_{t+1} + \big((\kappa-1)\upsilon_1 C_1(St_{t+1}) - \Theta_1(St_{t+1})\big)\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1}\end{aligned}$$

The nominal prices of the n-maturity bonds are linearly expressed as:

$$
\begin{aligned}
pr^{\$}_{n,t}(St_t) &= D^{\$}_{n,0}(St_t) + D^{\$}_{n,1}(St_t)Y_t \\
&= E_t(q_{t+1} - \pi_{t+1} + pr^{\$}_{n-1,t+1}(St_{t+1})) \\
&+ \frac{1}{2}Var_t(pr^{\$}_{n-1,t+1}(q_{t+1} - \pi_{t+1} + St_{t+1}))
\end{aligned}
$$

and the coefficients satisfy the following recursions:

$$
\begin{bmatrix} D^{\$}_{n,1}(1,:) \\ D^{\$}_{n,1}(2,:) \end{bmatrix} = \Delta \begin{bmatrix} (D^{\$}_{n-1,1}(1,:) - \Upsilon_1\Psi_1 \\ D^{\$}_{n-1,1}(2,:) - \Upsilon_1\Psi_1 \end{bmatrix} - \frac{1}{\omega}\begin{bmatrix} a_1 \\ a_1 \end{bmatrix}
$$

$$
\begin{aligned}
\begin{bmatrix} D^{\$}_{n,0}(1,:) \\ D^{\$}_{n,0}(2,:) \end{bmatrix} &= \Delta \begin{bmatrix} D^{\$}_{n-1,0}(1) - \Theta_0(1) + (\kappa-1)\upsilon_1 D_0(1) + \frac{1}{2}\Upsilon_{n-1,c}(1)\Upsilon_{n-1,c}(1)' \\ D^{\$}_{n-1,0}(2) - \Theta_0(2) + (\kappa-1)\upsilon_1 D_0(2) + \frac{1}{2}\Upsilon_{n-1,c}(2)\Upsilon_{n-1,c}(2)' \end{bmatrix} \\
&+ \begin{bmatrix} \Lambda_c(1) & \Lambda_c(2) \end{bmatrix}'
\end{aligned}
$$

$$
\begin{aligned}
\text{where,} \quad \Lambda_c(St_t) &= \kappa log\delta + (\kappa-1)(\upsilon_0 - C_0(St_t)) - \gamma\mu_c + \frac{1}{2}\gamma^2 a_1\Sigma\Sigma' a_1' \\
\Upsilon_{n-1,c}(St_t) &= (D^{\$}_{n-1,1}(St_t) + (\kappa-1)\upsilon_1 D_1(St_t))\Psi_2(St_t)\Sigma y(St_t)
\end{aligned}
$$

The nominal bond yield coefficients are:

$$
\begin{aligned}
C^{\$}_{n,0} &= -\frac{1}{n}D^{\$}_{n,0} \\
C^{\$}_{n,1} &= -\frac{1}{n}D^{\$}_{n,1}
\end{aligned}
$$

The one-period log return with an n-period nominal bond has the following solutions:

$$
\begin{aligned}
re^{\$}_{n,t+1} &= D^{\$}_{n-1,0}(St_{t+1}) - D^{\$}_{n,0}(St_t) + (D^{\$}_{n-1,1}(St_{t+1})\Psi_1(St_{t+1}) - D^{\$}_{n,1}(St_t))Y_t \\
&+ D^{\$}_{n-1,1}(St_{t+1})\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1}
\end{aligned}
$$

The excess one-period return is:

$$
\begin{aligned}
ry^{\$}_{n,t+1} &= D^{\$}_{n-1,0}(St_{t+1}) - D^{\$}_{n,0}(St_t) + (D^{\$}_{n-1,1}(St_{t+1})\Psi_1(St_{t+1}) - D^{\$}_{n,1}(St_t) \\
&+ D^{\$}_{1,1}(St_t)Y_t + D^{\$}_{n-1,1}(St_{t+1})\Psi_2(St_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1}
\end{aligned}
$$

From the asset pricing solutions, the expected excess one-period return with nominal bonds in each period can be further expressed as:

$$E(ry^{\$}_{n,t+1}|St_t = k) + \frac{1}{2}Var(ry^{\$}_{n,t+1}|St_t = k) = -Cov(q_{t+1}, ry^{\$}_{n,t+1}|St_t = k)$$
$$\approx -\Delta(k,:)\begin{bmatrix}\zeta(m,1)\\ \zeta(m,2)\end{bmatrix}$$

where,$\zeta(m,j) = \big((\kappa-1)\upsilon_1 A_1(j) - \Theta_1\big)\Psi_2(j)\Sigma y(j)\Sigma y^{'}(j)\Psi_2(j)^{'}(D_{n-1,1}(j)^{\$})^{'}$

A.13 L period ahead expectations

A vector X_{t+1} can be iterated forward recursively as:

$$X_{t+1} = \beta_0(St_{t+1}) + \beta_1(St_{t+1})Yt+1$$
$$Y_{t+1} = \Psi_1(St_{t+1})Y_t + \Psi_2(S_{t+1})\Sigma y(St_{t+1})\eta_{y,t+1}, \quad \eta_{y,t} \sim N(0,I)$$

$$E(X_{t+l}|St_t) = \underbrace{E(\beta_0(St_{t+l}|St_t)}_{\beta_0^l} + \underbrace{E(\beta_1(St_{t+l})\Psi_1(St_{t+l})\Psi_1(St_{t+l-1})....\Psi_1(St_{t+1})|St_t)Y_t}_{\beta_1^l}$$

where β are expressed as,

$$\beta_0^l(k) = \begin{bmatrix}\beta_0(1) & \beta_0(2)\end{bmatrix}\begin{bmatrix}p11 & p21\\ p12 & p22\end{bmatrix}^{l-1}\begin{bmatrix}p_{k1}\\ p_{k2}\end{bmatrix}$$

$$\beta_1^l(k) = \begin{bmatrix}\beta_1(1) & \beta_1(2)\end{bmatrix}\begin{bmatrix}p11\Psi_1(1) & p21\Psi_1(1)\\ p12\Psi_1(2) & p22\Psi_1(2)\end{bmatrix}^{l-1}\begin{bmatrix}\Psi_1(1) & 0\\ 0 & \Psi_1(2)\end{bmatrix}\begin{bmatrix}p_{k1}I\\ p_{k2}I\end{bmatrix}$$

A.14 Term Premium as function of one period return

As $y^{n,t} = -\frac{1}{n}pr_{n,t}$, the expression can be expanded in the form of an expected risk-free return, and the term premium shown below

$$
\begin{aligned}
y^{n,t} &= -\frac{1}{n} pr_{n,t} \\
&= \frac{1}{n}(\underbrace{-pr_{n,t} + pr_{n-1,t+1}}_{ry_{n,t+1}} - \underbrace{pr_{n-1,t+1} + pr_{n-2,t+2}}_{ry_{n-1,t+2}} ..... - pr^{1,t+n-1}) \\
y^{n,t} &= \underbrace{\frac{1}{n}(ry_{n,t+1} + ry_{n-1,t+2} + ... + y^{1,t+n-1})}_{Average\ of\ one\ period\ future\ holding\ return\ and\ the\ ultimate\ one\ period\ yield} \\
&= \underbrace{\frac{1}{n} E_t(rx_{n,t+1} + rx_{n-1,t+2} + ...)}_{Average\ of\ one\ period\ future\ excess\ return} + \underbrace{\frac{1}{n} E_t(y^{1,t} + y^{1,t+1} ..... + y^{1,t+n-1})}_{Average\ of\ one\ period\ future\ yield}
\end{aligned}
$$

$$
Term\ premium = \underbrace{\frac{1}{n} E_t(rx_{n,t+1} + rx_{n-1,t+2} + ...)}_{Average\ of\ one\ period\ future\ excess\ return}
$$

A.15 Source of Data used for training the asset pricing model

Real consumption growth is derived as the log first difference of the real private final consumption expenditure (PFCE) and is estimated quarterly using a rolling window of twelve months. Real PFCE data is sourced from the CEIC (OECD) database. Aggregate equity market data includes dividend per share and total return based on a value-weighted portfolio of firms listed in India's two major exchanges, the NSE and BSE. Market price and dividend information for these firms is obtained from Prowessdx. Log market returns and real dividend growth for the value-weighted portfolio are estimated quarterly with a rolling window of twelve months. The firms in the portfolio are screened based on the methodology outlined in the data library for Fama-French and Momentum Factors implemented for India as in Agarwalla et al. (2013). Zero coupon yields with maturity of one, two, five, and ten years are estimated using the Nelson-Siegel Svensson(NSS) parameters obtained from CCIL.

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